# A New 2×2 Coordinate Interleaved STBC for High-Rate Wireless Systems

Ertuğrul Başar and Ümit Aygölü
Istanbul Technical University, Faculty of Electrical & Electronics Engineering,
34469, Maslak, Istanbul, Turkey. {basarer,aygolu}@itu.edu.tr

***Abstract*— In this paper, we present a new full-rate, full-diversity coordinate interleaved design (CID) for multiple-input multiple-output (MIMO) systems with two transmit antennas. When compared with the conventional space-time block codes (STBCs) and coordinate interleaved orthogonal designs (CIODs), the proposed scheme is above the predefined full-rate limits since we ignore the pure orthogonality principle while transmitting extra symbols from available slots. The higher complexity of this structure due to non-orthogonality is reduced by using a simple conditional maximum-likelihood (ML) decoder. The new CID achieves full-diversity with a high minimum determinant, and outperforms its counterpart given by Sezginer and Sari with certain constellations, which make it to be considered for the future high data rate wireless applications.**

## I. Introduction

Alamouti's STBC [1] and spatial multiplexing (SM) are the two extreme cases for a MIMO system with two transmit antennas. Alamouti's STBC achieves full-diversity but it is only half-rate since it wastes available space-time slots while offering orthogonality. However, SM does not offer spatial diversity at the transmitter side but its data rate is twice as that of Alamouti's STBC. Some intermediate solutions to combine the attractive features of Alamouti's STBC and SM are recently proposed. One of them is the Golden code [2], which is known as one of the best 2×2 STBCs and actually exists in WiMAX standards, but the implementation of its complex sphere decoder is its main problem. In [3], Sezginer and Sari (SS) shown that a new generation STBC with reduced decoder complexity compared to the Golden code, which transmits four information symbols in four space-time slots, is possible without using a complex-structured sphere decoder. However, since the minimum determinant of this STBC is smaller than that of the Golden Code, this leads to a performance drawback. This paper presents a new design which achieves the Golden code's performance with SS-STBC's decoding complexity.

## II. The New Coordinate Interleaved Design

For two transmit antennas MIMO system, the CIOD given in [5], takes a block of two modulated symbols $x_0$ and $x_1$ and transmits them from two antennas in two time intervals according to the code matrix given by

$$\begin{bmatrix} x_{0R} + jx_{1I} & 0 \\ 0 & x_{1R} + jx_{0I} \end{bmatrix} \tag{1}$$

where $x_{iR}$ and $x_{iI}$ for $i = 0,1$, are the real and imaginary parts of $x_i$, columns and rows denote transmit antennas and time slots, respectively. The empty slots in (1) waste the available space-time slots for orthogonality, furthermore lead to a very high peak-to-average power ratio (PAPR) which prevents its practical implementation. We propose a new coordinate interleaved design which takes a block of four modulated symbols $x_0$, $x_1$, $x_2$, and $x_3$, and transmits them from two antennas in two time intervals according to the code matrix given by

$$\begin{bmatrix} a(x_{0R} + jx_{1I}) & c(x_{2R} + jx_{3I}) \\ d(x_{3R} + jx_{2I}) & b(x_{1R} + jx_{0I}) \end{bmatrix}. \tag{2}$$

In (2), $a$, $b$, $c$, and $d$ are the complex design parameters to be determined by the design criteria [4]. In terms of the equal total transmitted power in each symbol interval and for each symbol, the condition on $a$, $b$, $c$, and $d$ is given as $|a| = |b| = |c| = |d| = 1$. We named this code matrix as *coordinate interleaved design* (CID) instead of CIOD since it lost its pure orthogonality while transmitting additional symbols from empty slots of CIOD. For mathematical convenience we chose two receive antenna case as well as the proposed scheme can be easily extended to more than two transmit antennas. Let $\tilde{x}_0 = x_{0R} + jx_{1I}$, $\tilde{x}_1 = x_{1R} + jx_{0I}$, $\tilde{x}_2 = x_{2R} + jx_{3I}$, $\tilde{x}_3 = x_{3R} + jx_{2I}$ denote the interleaved symbols, then the received signals at two receive antennas are

$$\begin{aligned} r_0 &= h_{11} a\tilde{x}_0 + h_{12} c\tilde{x}_2 + n_0 \\ r_1 &= h_{11} d\tilde{x}_3 + h_{12} b\tilde{x}_1 + n_1 \\ r_2 &= h_{21} a\tilde{x}_0 + h_{22} c\tilde{x}_2 + n_2 \\ r_3 &= h_{21} d\tilde{x}_3 + h_{22} b\tilde{x}_1 + n_3 \end{aligned} \tag{3}$$

where $h_{ij}$ denotes the channel coefficient between the transmit antenna $j$ and receive antenna $i$, and $n_i$ represent the complex additive Gaussian noise at the receiver. Both of $h_{ij}$ and $n_i$ are i.i.d. complex Gaussian random variables with the pdfs $N_{\mathbb{C}}(0,1)$ and $N_{\mathbb{C}}(0, N_0)$, respectively. The ML detector makes a search over all possible values of $x_0$, $x_1$, $x_2$, and $x_3$ and decides in favor of the quadruple which minimizes the decision metric,

$$\begin{aligned} D(x_0, x_1, x_2, x_3) = {} & |r_0 - h_{11} a\tilde{x}_0 - h_{12} c\tilde{x}_2|^2 + |r_1 - h_{11} d\tilde{x}_3 - h_{12} b\tilde{x}_1|^2 \\ & + |r_2 - h_{21} a\tilde{x}_0 - h_{22} c\tilde{x}_2|^2 + |r_3 - h_{21} d\tilde{x}_3 - h_{22} b\tilde{x}_1|^2 . \end{aligned} \tag{4}$$

This search requires the computation of $M^4$ metrics, however by computing intermediate signals we reduce the decoder complexity from $M^4$ to $2M^3$ as follows. From (3), we compute intermediate signals for all possible values of $x_2$ and $x_3$ and while going over this search, for only the correct values of $\tilde{x}_2$, therefore $\tilde{x}_3$, we obtain the intermediate signals as

978-0-7695-3393-3/08 $25.00 © 2008 IEEE
DOI 10.1109/WiMob.2008.15

$$
\begin{aligned}
\tilde{z}_0 &= r_0 - h_{12}c\tilde{x}_2 = h_{11}a\tilde{x}_0 + n_0 \\
\tilde{z}_1 &= r_1 - h_{11}d\tilde{x}_3 = h_{12}b\tilde{x}_1 + n_1 \\
\tilde{z}_2 &= r_2 - h_{22}c\tilde{x}_2 = h_{21}a\tilde{x}_0 + n_2 \\
\tilde{z}_3 &= r_3 - h_{21}d\tilde{x}_3 = h_{22}b\tilde{x}_1 + n_3
\end{aligned}
\tag{5}
$$

then we combine them to form $\tilde{y}_0 = \left(h_{11}^*\tilde{z}_0 + h_{21}^*\tilde{z}_2\right)/a$ and $\tilde{y}_1 = \left(h_{12}^*\tilde{z}_1 + h_{22}^*\tilde{z}_3\right)/b$. It can be proved that, by using the ML decoding procedures for CIOD given in (2), ML estimates of $x_0$ and $x_1$ conditioned on the pair ($x_2$, $x_3$) are found as

$$
\begin{aligned}
x_0^{ML} = \arg\min_{x_0} \Big\{ & \left(|h_{12}|^2 + |h_{22}|^2\right)\left|\hat{x}_{0R} - \left(|h_{11}|^2 + |h_{21}|^2\right)x_{0R}\right|^2 \\
& + \left(|h_{11}|^2 + |h_{21}|^2\right)\left|\hat{x}_{0I} - \left(|h_{12}|^2 + |h_{22}|^2\right)x_{0I}\right|^2 \Big\}
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
x_1^{ML} = \arg\min_{x_1} \Big\{ & \left(|h_{11}|^2 + |h_{21}|^2\right)\left|\hat{x}_{1R} - \left(|h_{12}|^2 + |h_{22}|^2\right)x_{1R}\right|^2 \\
& + \left(|h_{12}|^2 + |h_{22}|^2\right)\left|\hat{x}_{1I} - \left(|h_{11}|^2 + |h_{21}|^2\right)x_{1I}\right|^2 \Big\}.
\end{aligned}
\tag{7}
$$

where $\hat{x}_0 = \mathrm{Re}\{\tilde{y}_0\} + j\,\mathrm{Im}\{\tilde{y}_1\}$ and $\hat{x}_1 = \mathrm{Re}\{\tilde{y}_1\} + j\,\mathrm{Im}\{\tilde{y}_0\}$. The minimizations in (6) and (7) are over all possible values of the transmitted symbols $x_0$ and $x_1$ taken from $M$-component signal set. Then we minimize the metric $D\left(x_0^{ML}, x_1^{ML}, x_2, x_3\right)$ over all possible values of the pair ($x_2$, $x_3$) instead of minimizing $D\left(x_0, x_1, x_2, x_3\right)$ over all possible values of $x_0$, $x_1$, $x_2$, and $x_3$. This leads to a reduction in decoder complexity from $M^4$ to $2M^3$. In other words, instead of suffering from $M^4$ metric computations, we only search with a decoding complexity of $M^2$, and obtain conditional ML estimates of $x_0$ and $x_1$, which needs an additional decoding complexity of $2M$ per each step of $M^2$ calculation. Therefore, we obtain a total decoding complexity of $2M \times M^2 = 2M^3$ which is the same as that of SS-STBC [3].

## III. Performance Issues and Simulation Results

For a CID, the considered parameter to obtain full-diversity and high coding gain is the coordinate product distance (*CPD*). The optimum rotation angles which give maximum *CPD*s for QPSK with symbols on the two axis and *M*-QAM having odd-integer coordinates are found as equal to 13.2885º and 31.7175º, respectively. These optimum angles are used for the new CID because of its coordinate interleaved symbols. The complex design parameters *a*, *b*, *c*, and *d* in (2) are used to obtain full-diversity and high coding gain. Therefore, we aim to obtain non-vanishing determinant for all realizations of the distance matrix by optimizing design parameters *a*, *b*, *c*, and *d*. *The sufficient condition for a non-vanishing determinant is found as* $abc^*d^* = -a^*b^*cd = \pm j$. In our simulations we use the set $(j,1,1,1)$. By this condition, at the optimal rotation angle for a given signal constellation, the new CID given in (2) achieves the same minimum determinant as that of CIOD given in (1). Table 1 shows the minimum determinants of the Golden Code [2], SS-STBC [3], CIOD [5], and the new CID for QPSK, 4/16/64-QAM modulations. For all modulations, CIOD and the new CID use rotated signal constellations to obtain maximum *CPD*. From these results we conclude that the new CID provides the same minimum determinant as that of the Golden Code and higher minimum determinant as that of SS-STBC for all modulations. In Fig. 1, we compare the codeword error performances (CER) of the SS-STBC and the new CID, as a function of received SNR. As seen from Fig. 1, the proposed CID outperforms SS-STBC for 4-QAM and 16-QAM constellations according to the minimum determinants given in Table 1.

TABLE 1: Minimum Determinants for Golden Code [2], SS-STBC [3], CIOD (1), and new CID (2)

| Min. Det. | QPSK | 4-QAM | 16-QAM | 64-QAM |
|---|---|---|---|---|
| Golden C. | 0.8 | 3.2 | 3.2 | 3.2 |
| SS-STBC | 0.5 | 2 | 1.98 | 1.88 |
| CIOD | 0.7998 | 3.2 | 3.2 | 3.2 |
| New CID | 0.7998 | 3.2 | 3.2 | 3.2 |

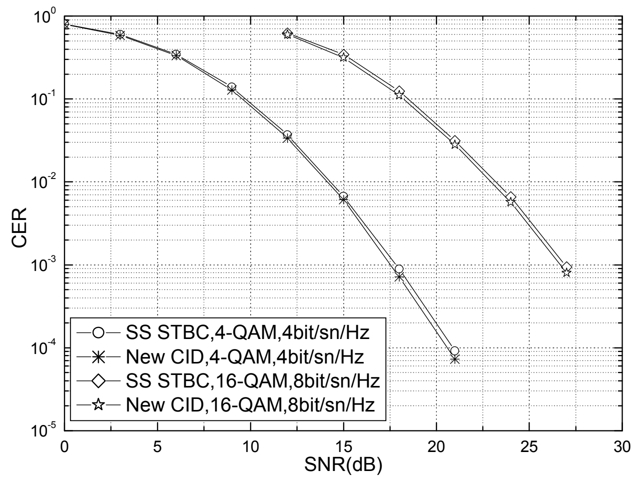


Fig. 1: Codeword error performances of SS-STBC and new CID for 2×2 MIMO system

## References

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998

[2] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The Golden code: a $2\times 2$ full-rate space-time code with non-vanishing determinants,", *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1432-1436, Apr. 2005.

[3] S. Sezginer and H. Sari, "Full-rate full-diversity $2\times 2$ space-time codes of reduced decoder complexity," *IEEE Commun. Letters*, vol. 11, no. 12, pp. 973-975, Dec. 2007.

[4] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744-765, Mar. 1998.

[5] M. Z. A. Khan and B. S. Rajan, "Single-symbol maximum likelihood decodable linear STBCs," *IEEE Trans. Inform. Theory*, vol. 52, pp. 2062-2091, May 2006.