Full-rate full-diversity STBCs for three and four transmit antennas

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A novel technique is presented for the construction of full-rate, full-diversity space–time block codes (STBCs) from orthogonal STBCs (OSTBCs), having empty slots left in their codeword matrices for orthogonality. Two new STBCs are obtained, which are both full-rate and full-diversity, for three and four transmit antennas. The higher decoding complexity of these structures is reduced owing to non-orthogonality by using a conditional maximum-likelihood decoder. The new optimised codes provide better error performance than their full-rate full-diversity counterparts given in the literature.

Introduction: Orthogonal STBCs (OSTBCs) are one of the most attractive space–time coding techniques for exploiting the spatial diversity of a MIMO fading channel [1]. They allow low-complexity ML detection; however, their code rate is low for more than two transmit antennas. Space–time block codes (STBCs) using co-ordinate interleaved orthogonal designs (CIODs) proposed in [2] allow single-symbol maximum likelihood (ML) decoding and offer higher data rates than OSTBCs. However, contrary to OSTBCs, CIODs may not achieve full-diversity with the conventional constellations such as PSK or QAM. To achieve full-diversity, signal constellations must be rotated by an angle. In this Letter, we present a novel approach to construct fast decodable, full-rate, full-diversity STBCs from OSTBCs given in the literature and give two new designs for three and four transmit antennas.

System model: We consider an $n \times m$ MIMO system where n and m are the number of transmit and receive antennas, respectively. At each signalling interval t, $t = 1, 2, \ldots, l$, c_t^i, $i = 1, 2, \ldots, n$ are simultaneously transmitted from n antennas through the quasistatic flat fading channel with path gain from transmit antenna i to receive antenna j denoted by h_{ij}. The path gains are assumed to be independent complex Gaussian random variables with variance 0.5 per dimension. As a result, the received signal at time t and receive antenna j is given as

$$r_t^j = \sum_{i=1}^{n} h_{i,j} c_t^i + n_t^j \tag{1}$$

where the noise samples n_t^j are zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension. Assuming that perfect channel state information (CSI) is available, the receiver minimises the ML decision metric

$$\sum_{t=1}^{l} \sum_{j=1}^{m} \left| r_t^j - \sum_{i=1}^{n} h_{i,j} c_t^i \right|^2 \tag{2}$$

over all possible symbols c_t^i. For the case of ML decoding, we denote the total number of metric computations, γ, as the decoding complexity of the corresponding STBC. Decoding complexity γ of an STBC, which transmits κ information symbols during $n \times l$ space-time slots, cannot exceed M^{κ} where M is the size of signal constellation. In this work, we try to reduce this decoder complexity from M^{κ} into an acceptable level.

Design procedure: Let $\boldsymbol{Q}_n$ denote a classical complex orthogonal design for n transmit antennas such as those given in [1], which transmits κ information symbols having empty slots left in its codeword matrix for orthogonality; we obtain a full-rate, full-diversity STBC $\boldsymbol{X}_n$ from $\boldsymbol{Q}_n$ as

$$\boldsymbol{X}_n = \boldsymbol{Q}_n + \boldsymbol{PG} \tag{3}$$

where $\boldsymbol{G}$ is the codeword matrix with λ additional information symbols to be transmitted from empty slots of $\boldsymbol{Q}_n$. Since the matrix $\boldsymbol{X}_n$ is non-orthogonal due to the added matrix $\boldsymbol{G}$, an optimising diagonal matrix $\boldsymbol{P}$ with complex entries is introduced in (3) to make $\boldsymbol{X}_n$ full-rank and further to maximise its minimum determinant. $\boldsymbol{Q}_n$ and $\boldsymbol{PG}$ contain non-overlapping entries. Owing to the non-orthogonal structure of $\boldsymbol{X}_n$, by direct computation, $M^{(\kappa+\lambda)}$ metric computations are required for ML decoding. When compared with the decoding complexity of $\boldsymbol{Q}_n$, which is κM, this increase in complexity is unacceptable. However, we try to eliminate the terms transmitted from empty slots of $\boldsymbol{Q}_n$, which comes from $\boldsymbol{PG}$ and causes a non-orthogonal structure, by computing intermediate signals from the received signals for all possible values of the additional symbols $x_{\kappa+1}, x_{\kappa+2}, \ldots, x_{\kappa+\lambda}$; then we use the same decoding procedure for $\boldsymbol{Q}_n$ to obtain conditional ML estimates of $x_1, x_2, \ldots, x_{\kappa}$. Finally, we minimise the decision metric given in (2) for $x_1^{ML}, x_2^{ML}, \ldots, x_{\kappa}^{ML}, x_{\kappa+1}, x_{\kappa+2}, \ldots, x_{\kappa+\lambda}$. In other words, instead of searching over all possible values of $x_1, x_2, \ldots, x_{\kappa+\lambda}$, and suffering from $M^{\kappa+\lambda}$ metric computations, we only search with a decoding complexity of M^{λ}, and obtain conditional ML estimates of $x_1, x_2, \ldots, x_{\kappa}$, which needs an additional decoding complexity of κM per each step of M^{λ} calculation. Therefore, we obtain a total decoding complexity of $\gamma = \kappa M \times M^{\lambda} = \kappa M^{\lambda+1}$.

Let us consider the following OSTBC for four transmit antennas:

$$\boldsymbol{Q}_4 = \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ x_3^* & 0 & -x_1^* & x_2 \\ 0 & x_3^* & -x_2^* & -x_1 \end{bmatrix} \tag{4}$$

In accordance with our definition given in (3) we propose the following full-rate STBC:

$$\boldsymbol{X}_4 = \begin{bmatrix} x_1 & x_2 & x_3 & ax_4 \\ -x_2^* & x_1^* & bx_4^* & x_3 \\ x_3^* & cx_4^* & -x_4^* & x_2 \\ dx_4^* & x_3^* & -x_2^* & -x_1 \end{bmatrix} \tag{5}$$

where a, b, c, and d are the entries of the diagonal matrix $\boldsymbol{P}$ to be determined by the rank and determinant criteria [3]. The decoding procedure for (5) can be summarised as follows.

From the received signals $r_t^j, j = 1,2, \ldots, m$, we eliminate the effect of the additional symbol x_4 in (5) by computing the intermediate signals for all possible values of it as

$$z_t{}^j = r_t{}^j - h_{k,j} \beta_{t,k}(x_4) \tag{6}$$

where $\beta_{t,k}(x_4)$ represents the codeword element at the tth row and kth column of $\boldsymbol{X}_4$, which is the extra transmitted symbol x_4 or its conjugate $x_4{}^*$. Then, by considering the orthogonal nature of $\boldsymbol{Q}_4$, we obtain conditional ML estimates for x_1, x_2 and x_3 as

$$\begin{aligned} x_1^{ML} &= \sum_{j=1}^{m} \left(z_1{}^j h_{1,j}^* + (z_2^j)^* h_{2,j} - (z_3^j)^* h_{3,j} - z_4^j h_{4,j}^* \right) \\ x_2^{ML} &= \sum_{j=1}^{m} \left(z_1{}^j h_{2,j}^* - (z_2{}^j)^* h_{1,j} + z_3{}^j h_{4,j}^* - (z_4{}^j)^* h_{3,j} \right) \\ x_3^{ML} &= \sum_{j=1}^{m} \left(z_1{}^j h_{3,j}^* + z_2{}^j h_{4,j}^* + (z_3{}^j)^* h_{1,j} + (z_4{}^j)^* h_{2,j} \right) \end{aligned} \tag{7}$$

We obtain a total decoding complexity of $3M^2$ by minimising decision statistics for $x_1^{ML}, x_2^{ML}, x_3^{ML}, x_4$ instead of M^4 metric computations. By removing the rightmost column of (5) we obtain the new STBC for three transmit antennas, which can be expressed in the form of (3). ML decoding procedure for this new STBC is the same as that described above for $\boldsymbol{X}_4$ when taking $h_{4,j}$ in (6) and (7) as equal to zero.

STBCs for quasistatic fading channels are designed according to rank and determinant criteria [3], to maximise diversity gain G_d and coding gain G_c, respectively. Complex design parameters a, b, c, and d in (5) are used to obtain full-diversity and high coding gain. In terms of the equal total transmitted power in each symbol interval and for each symbol, the constraint on a, b, c, and d is given as $|a| = |b| = |c| = |d| = 1$. Thanks to the special forms of the proposed STBCs, after an exhaustive computer search, by setting these parameters as $a = j$, $b = c = d = \sin 30^\circ + j\cos 30^\circ$ for QPSK with symbols on the two axes, and as $a = b = c = d = \sin 30^\circ + j\cos 30^\circ$ for M-QAM having odd integer co-ordinates, we obtain full-diversity with the same minimum determinants as for OSTBCs, namely, both $\boldsymbol{Q}_4$ and $\boldsymbol{X}_4$ have minimum determinant values of 16 and 256, OSTBC and the new STBC for three transmit antennas both have minimum determinant values of 8 and 64 for QPSK and $\boldsymbol{M}$-QAM, respectively.

Simulation results: QPSK symbol error rate (SER) curves of the OSTBC, the new STBC and CIOD [2] for a 4×2 MIMO system operating on a quasistatic Rayleigh fading channel are shown in Fig. 1 as a function of E_s/N_0, where E_s denotes the average transmitted signal energy per symbol. From these results, we conclude that the new design outperforms full-rate CIOD with increasing SNR values. For an SER value of 10^{-5}, the new design provides 0.25 dB advantage over CIOD. However, as expected, both CIOD and the new design are outperformed by OSTBC, but the slopes of these three curves are the same. Performance curves for OSTBC, the new STBC and CIOD for a 3×2 MIMO system are also shown in Fig. 1. From these curves, we see that the new design outperforms CIOD for all SNR values. For example, at the SER value of 10^{-4}, the new design provides 0.8 dB advantage over CIOD.

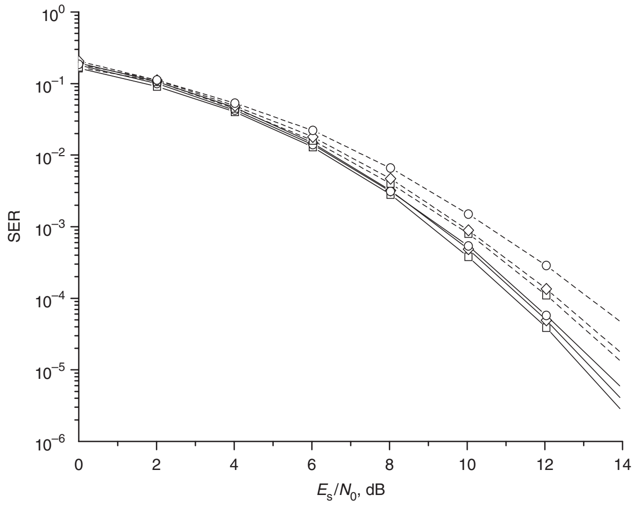

Fig. 1 *QPSK SER performance comparisons*

—□— 4Tx and OSTBC
—◇— 4Tx and new STBC
—○— 4Tx and CIOD
---□--- 3Tx and OSTBC
---◇--- 3Tx and new STBC
---○--- 3Tx and CIOD

Conclusions: We present a novel design technique for the construction of full-rate, full-diversity STBCs with reduced ML decoder complexity from classical OSTBCs. We have given two specific examples with two new STBCs for three and four transmit antennas. We have also optimised the proposed schemes to obtain best performance in the context of full-diversity and maximum coding gain. When compared with the previous most powerful full-rate, full-diversity STBCs from CIODs, the new scheme provides better error performance.

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