High-Rate Full-Diversity Space-Time Block Codes with Linear Receivers

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***Abstract* —In this paper, we deal with the design of high-rate space-time block codes (STBCs) that achieve full-diversity with linear receivers which enable symbol-wise decoding. We propose three new high-rate coordinate interleaved STBCs and prove that they can achieve full-diversity with linear receivers for any optimally rotated square QAM constellation. Recently, Shang and Xia proved that the symbol rate of an STBC achieving full-diversity with a linear receiver is upper bounded by one complex information symbol per channel use (pcu). However, we show that with the use of coordinate interleaving, the proposed STBCs can exceed this upper bound to 4/3 complex information symbols pcu for two, three and four transmit antennas. For the symbol-by-symbol decoding of the proposed STBCs, we adapt the partial interference cancellation (PIC) group decoding algorithm recently proposed by Guo and Xia, and then further modify this decoder by applying successive interference cancellation (SIC) operation. Simulation results show that when linear receivers with minimum decoding complexity are used, the proposed STBCs achieve better error performance than their counterparts given in the literature.**

I. Introduction

Space-time block codes (STBCs) have been comprehensively studied since the early works in [1] and [2]. In [1], Alamouti proposed a remarkable scheme for MIMO systems with two transmit antennas, which allows a low-complexity maximum likelihood (ML) decoder. Then, STBCs for more than two transmit antennas were designed in [2]. For such codes, the ML decoding can be performed in symbol-wise way due to the orthogonality of their code matrix. However, it has been proved that the symbol rate of an *orthogonal STBC* (OSTBC) is upper bounded by 3/4 (rate-3/4) complex information symbols per channel use (pcu) for more than two transmit antennas [3]. The orthogonality condition was then relaxed by *quasi-orthogonal STBCs* (QOSTBCs) to exceed this upper bound at the expense of increased decoding complexity [4]. QOSTBCs were then modified to obtain full transmit diversity with constellation rotation [5]. Besides the QOSTBCs, a special class of OSTBCs named *coordinate interleaved orthogonal designs* (CIODs) which exceed the upper bound mentioned above, having an ML decoder with linear decoding complexity, were proposed in [6]. Later, several high-rate STBCs were introduced in [7,8], however their ML decoding complexities grow exponentially with the constellation size which make their implementation difficult and expensive.

The *full rank criterion* derived in [9] ensures maximum diversity order in a quasi-static Rayleigh fading channel. However, this criterion holds for the optimal ML decoder whose implementation becomes infeasible in some cases due to its high computational complexity. To reduce this higher decoding complexity, one may prefer suboptimum decoding algorithms such as zero-forcing (ZF) or minimum mean squared error (MMSE) estimation to perform symbol-wise decoding with linear complexity [10]. However, in such cases, the full rank criterion cannot guarantee full transmit diversity. For OSTBCs, symbol-wise decoding is equivalent to the ML decoding, therefore, full-diversity can be achieved with linear receivers. Recently, some researchers are focused on full-diversity non-orthogonal STBCs which allow symbol-wise decoding. Two classes of STBCs named *Toeplitz codes* and *Overlapped Alamouti codes* were proposed by Zhang, et. al. [11] and Shang and Xia [12], respectively. It has been proved in [13] that the symbol rate of an STBC achieving full-diversity with a linear receiver is upper bounded by 1 symbol pcu. By generalizing the works in [11-13], for general linear dispersive STBCs [14], Guo and Xia proposed a novel decoding scheme in [15] called *partial interference cancellation* (PIC) *group decoding*. The main idea of the PIC group decoding algorithm is to divide the information symbols in an STBC into several groups and decode these groups independently after PIC group decoding algorithm is applied. According to the full-diversity criteria in [15], two new STBCs were proposed for two and four transmit antennas with symbol rates 4/3, however, due to their structure, these STBCs require the detection of two complex symbols jointly, which corresponds to a higher decoding complexity than that of single-symbol decodable STBCs with linear receivers.

In this paper, we deal with the design of single-symbol decodable, high-rate, full-diversity STBCs. Our contributions in this paper are given as below:

- We propose a novel high-rate full-diversity coordinate interleaved STBC structure. Using this structure, we introduce three new rate-4/3 STBCs for two, three and four transmit antennas.
- We formulate a single-symbol PIC decoder, which decomposes the embedded symbols of the proposed STBCs into independent groups each formed by real and imaginary parts of a single complex information symbol, then decodes these groups (symbols) separately. We choose PIC decoder since it provides an intermediate solution between error performance and complexity.
- We prove that the proposed STBCs can achieve full-diversity with linear receivers for any optimally rotated square *M*-QAM constellation.

- In accordance with the special structure of the proposed STBC design, we further modify this single-symbol PIC decoder with *successive interference cancellation* (SIC) operation and obtain a novel PIC-SIC-ML decoder.
- We show by computer simulation results that the proposed STBCs achieve better error performance than their counterparts given in the literature when linear receivers are used.

Notations: Bold, lowercase and capital letters are used for column vectors and matrices, respectively. $(.)^T$ and $(.)^H$ denote transposition and Hermitian transposition, respectively. For a complex variable x, x_R and x_I denote the real and imaginary parts of x, i.e., $x = x_R + jx_I$, where $j = \sqrt{-1}$. The fields of real and complex numbers are denoted by $\mathbb{R}$ and $\mathbb{C}$, respectively. χ represents a complex signal constellation. $\mathbf{I}_m$ and $\mathbf{0}_{(m\times n)}$ denote the $m\times m$ identity matrix and the $m\times n$ matrix with all zero elements, respectively. The Euclidean norm of a vector is denoted by $\|.\|$.

II. Channel Model

Let us consider an $n_T\times n_R$ quasi-static Rayleigh flat fading MIMO channel, where n_T and n_R denote the number of transmit and receive antennas, respectively. The received $T\times n_R$ signal matrix $\mathbf{Y}\in\mathbb{C}^{T\times n_R}$ can be modeled as

$$\mathbf{Y} = \mathbf{XH} + \mathbf{N} \tag{1}$$

where $\mathbf{X}\in\mathbb{C}^{T\times n_T}$ is the codeword (transmission) matrix, transmitted over T channel uses. $\mathbf{H}$ and $\mathbf{N}$ are the $n_T\times n_R$ channel matrix and the $T\times n_R$ noise matrix, respectively. The entries of $\mathbf{H}$ and $\mathbf{N}$ are i.i.d. complex Gaussian random variables with the pdfs $N_{\mathbb{C}}(0,1)$ and $N_{\mathbb{C}}(0,N_0)$, respectively. We assume, $\mathbf{H}$ remains constant during the transmission of a codeword, and take independent values from one codeword to another. The realization of $\mathbf{H}$ is assumed to be known at the receiver, but not at the transmitter. In order to apply operations for extracting and decoding the transmitted information symbols from $\mathbf{Y}$, the channel model in (1) must be rewritten as [14]

$$\mathbf{y} = \mathcal{H}\mathbf{x} + \mathbf{n} \tag{2}$$

where $\mathbf{y}\in\mathbb{R}^{2Tn_R}$ is the received signal vector, $\mathcal{H}\in\mathbb{R}^{2Tn_R\times 2K}$ is the equivalent channel matrix, $\mathbf{x} = \left[x_{0R}, x_{0I}, \ldots, x_{(K-1)R}, x_{(K-1)I}\right]^T$ is the real information symbol vector; $\mathbf{n}\in\mathbb{R}^{2Tn_R}$ is the additive Gaussian noise vector having i.i.d entries with the pdf $N_{\mathbb{R}}(0, N_0/2)$.

Definition 1: (Symbol Rate) The symbol rate of an STBC with the codeword matrix $\mathbf{X}$ is defined as $R = K/T$ symbols per channel use where K is the number of information symbols embedded in $\mathbf{X}$. An STBC is said to be high-rate if $R > 1$.

Definition 2: (Decoding Complexity) The decoding complexity is the number of metric computations performed to decode the information symbol vector $\mathbf{x}$. By direct approach, ML decoding of $\mathbf{x}$ is performed by deciding in favor of the symbol vector which minimizes the following metric

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\chi^K} \|\mathbf{y} - \mathcal{H}\mathbf{x}\|^2 . \tag{3}$$

For a signal constellation of size M, the minimization in (3) requires the computation of M^K metrics which is the worst-case detection complexity since all the symbols in $\mathbf{X}$ are detected jointly. On the other hand, in case of single-symbol decoding, the total decoding complexity becomes *linear* (KM) since all symbols in $\mathbf{X}$ are decoded separately.

III. New Coordinate Interleaved STBCs

In this section, we start by the definition of CIOD, then inspiring from CIODs, we present our high-rate coordinate interleaved STBC structure and give design examples for two, three and four transmit antennas. After the formulation of the proposed STBC structure, we give its single-symbol decoding algorithm which is adapted from the PIC group decoding algorithm recently proposed by Guo and Xia [15].

Definition 3: A CIOD of size $n_T\times n_T$ with symbols x_l, $l = 0,1,\ldots,K-1$ (where K is even) is given as [6]

$$\begin{bmatrix} \Theta(\tilde{x}_0, \tilde{x}_1, \ldots, \tilde{x}_{K/2-1}) & \mathbf{0}_{(n_T/2)\times(n_T/2)} \\ \mathbf{0}_{(n_T/2)\times(n_T/2)} & \Theta(\tilde{x}_{K/2}, \tilde{x}_{K/2+1}, \ldots, \tilde{x}_{K-1}) \end{bmatrix} \tag{4}$$

where Θ is the complex orthogonal design (COD) of size $(n_T/2)\times(n_T/2)$ with symbol rate K/n_T ; $\tilde{x}_i = \mathrm{Re}\{x_i\}$ $+ j\,\mathrm{Im}\{x_{(i+K/2)_K}\}$ and $(a)_K$ denotes a mod K. It is shown in [6] that $R = 1$ CIOD exists if and only if $n_T = 2$ and 4. $R = 1$ CIOD can also be generalized for $n_T = 3$.

Although CIODs can be decoded with linear decoding complexity, symbol rate-1 may not be sufficient for next generation wireless communication systems. Therefore, it is desirable to achieve higher symbol rates than 1 with linear receivers. However, it has been proved in [13] that the symbol rate of an STBC achieving full diversity with linear receiver is upper-bounded by 1. Note that the upper bound in symbol rates for OSTBCs (which is 3/4 for $n_T > 2$) is exceeded by CIODs. Similarly, coordinate interleaved structures compromising orthogonality allow achieving higher symbol rates than 1 with linear receivers while ensuring full-diversity. We propose the following high-rate full-diversity STBC structure,

$$\begin{bmatrix} \Theta(\tilde{x}_0, \tilde{x}_1, \ldots, \tilde{x}_{K/2-1}) & \mathbf{0}_{(n_T/2)\times(n_T/2)} \\ \Theta(\tilde{x}_K, \tilde{x}_{K+1}, \ldots, \tilde{x}_{3K/2-1}) & \Theta(\tilde{x}_{K/2}, \tilde{x}_{K/2+1}, \ldots, \tilde{x}_{K-1}) \\ \mathbf{0}_{(n_T/2)\times(n_T/2)} & \Theta(\tilde{x}_{3K/2}, \tilde{x}_{3K/2+1}, \ldots, \tilde{x}_{2K-1}) \end{bmatrix} \tag{5}$$

where Θ is as defined in (4) and

$$\tilde{x}_i = \begin{cases} \mathrm{Re}\{x_i\} + j\,\mathrm{Im}\{x_{(i+K/2)_K}\}, & \text{for } 0 \le i \le K-1 \\ \mathrm{Re}\{x_i\} + j\,\mathrm{Im}\{x_{(i+K/2)_K+K}\}, & \text{for } K \le i \le 2K-1. \end{cases} \tag{6}$$

As seen from (5), the symbol rate of the proposed STBC is $4K/3n_T$. We conclude that if the symbol rate of the CIOD of (4) for n_T transmit antennas is $R = K/n_T$, then the symbol rate of the corresponding new STBC for n_T transmit antennas is $4R/3$. According to (5), we propose the following rate-4/3

STBCs for two and four transmit antennas, respectively as

$$\begin{bmatrix} x_{0R}+jx_{1I} & 0 \\ x_{2R}+jx_{3I} & x_{1R}+jx_{0I} \\ 0 & x_{3R}+jx_{2I} \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} x_{0R}+jx_{2I} & x_{1R}+jx_{3I} & 0 & 0 \\ -x_{1R}+jx_{3I} & x_{0R}-jx_{2I} & 0 & 0 \\ x_{4R}+jx_{6I} & x_{5R}+jx_{7I} & x_{2R}+jx_{0I} & x_{3R}+jx_{1I} \\ -x_{5R}+jx_{7I} & x_{4R}-jx_{6I} & -x_{3R}+jx_{1I} & x_{2R}-jx_{0I} \\ 0 & 0 & x_{6R}+jx_{4I} & x_{7R}+jx_{5I} \\ 0 & 0 & -x_{7R}+jx_{5I} & x_{6R}-jx_{4I} \end{bmatrix}. \quad (8)$$

It should be noted that by removing the rightmost column of (8), we obtain a rate-4/3 STBC for three transmit antennas.

For decoding operation of (5), we adapt the PIC group decoding technique to our STBCs with the real equivalent channel model given in (2). Suppose the new STBC of (5) transmits $2K$ complex information symbols drawn from a rotated M-QAM constellation. In this case, we rewrite the $2Tn_R \times 4K$ real equivalent channel matrix in (2) as

$$\mathcal{H}=\begin{bmatrix} \mathbf{h}_{0,R} & \mathbf{h}_{0,I} & \mathbf{h}_{1,R} & \mathbf{h}_{1,I} & \cdots & \mathbf{h}_{(2K-1),R} & \mathbf{h}_{(2K-1),I} \end{bmatrix} \quad (9)$$

where $\mathbf{h}_{i,R}$ and $\mathbf{h}_{i,I}$ for $i=0,1,\ldots,2K-1$ denote the columns of $\mathcal{H}$ corresponding to the transmission of the real and imaginary parts of the complex information symbol x_i. If we define the $2Tn_R \times 2$ matrices

$$\mathcal{H}_i=\begin{bmatrix} \mathbf{h}_{i,R} & \mathbf{h}_{i,I} \end{bmatrix},\ i=0,1,\ldots 2K-1$$

which corresponds to the symbol x_i, then we can rewrite $\mathcal{H}$ as

$$\mathcal{H}=[\mathcal{H}_0 \quad \mathcal{H}_1 \quad \cdots \quad \mathcal{H}_{2K-1}]. \quad (10)$$

Let us present x_i in the vector form as $\mathbf{x}_i=\begin{bmatrix} x_{iR} & x_{iI}\end{bmatrix}^T$ for $i=0,1,\ldots 2K-1$ from where we can rewrite (2) as

$$\mathbf{y}=\sum_{i=0}^{2K-1}\mathcal{H}_i\mathbf{x}_i+\mathbf{n}. \quad (11)$$

Suppose we want to decode the k-th complex symbol x_k, or equivalently the vector $\mathbf{x}_k$. We use the PIC group decoding algorithm to completely eliminate the interferences coming from other symbols as follows. First we form the matrix $\mathcal{H}_k^c \in \mathbb{R}^{2Tn_R\times(2K-2)}$ by removing the columns belonging to $\mathcal{H}_k$ from $\mathcal{H}$ as

$$\mathcal{H}_k^c=[\mathcal{H}_0 \quad \mathcal{H}_1 \quad \cdots \quad \mathcal{H}_{k-1} \quad \mathcal{H}_{k+1} \quad \cdots \quad \mathcal{H}_{2K-1}]. \quad (12)$$

Then we obtain the projection matrix $\mathbf{Q}_k \in \mathbb{C}^{2Tn_R\times 2Tn_R}$ using $\mathcal{H}_k^c$ as [15]

$$\mathbf{Q}_k=\mathcal{H}_k^c\left(\left(\mathcal{H}_k^c\right)^T\mathcal{H}_k^c\right)^{-1}\left(\mathcal{H}_k^c\right)^T. \quad (13)$$

In (13), we assume $\mathcal{H}_k^c$ is full-column rank, otherwise $\mathbf{Q}_k$ cannot be calculated. The minimum number of receive antennas must be two for the proposed STBCs to ensure that $\mathcal{H}_k^c$ is full-column rank. Finally we obtain the projection matrix $\mathbf{P}_k \in \mathbb{C}^{2Tn_R\times 2Tn_R}$ for which $\mathbf{P}_k\mathcal{H}_k^c=\mathbf{0}$, from

$$\mathbf{P}_k=\mathbf{I}_{2Tn_R}-\mathbf{Q}_k. \quad (14)$$

Therefore, multiplying the received signal vector by $\mathbf{P}_k$, all interferences from the other information symbols are canceled. Let $\mathbf{z}_k \triangleq \mathbf{P}_k\mathbf{y}$, then using the fact that $\mathbf{P}_k\mathcal{H}_k^c=\mathbf{0}$, we obtain

$$\begin{aligned}\mathbf{z}_k &= \mathbf{P}_k\sum_{i=0}^{2K-1}\mathcal{H}_i\mathbf{x}_i+\mathbf{P}_k\mathbf{n} \\ &= \mathbf{P}_k\mathcal{H}_k\mathbf{x}_k+\mathbf{P}_k\mathbf{n}.\end{aligned} \quad (15)$$

Although the noise term in (15) is no longer white Gaussian, it is also proved that minimum Euclidean distance can be used for ML decoding of $\mathbf{x}_k$ as follows [15]

$$\hat{\mathbf{x}}_k^{PIC}=\arg\min_{\mathbf{x}\in\chi}\left\|\mathbf{z}_k-\mathbf{P}_k\mathcal{H}_k\mathbf{x}\right\| \quad (16)$$

where $\mathbf{x}=\begin{bmatrix} x_R & x_I\end{bmatrix}^T$. In other words, Eq. (15) can be viewed as $\mathbf{x}_k$ is transmitted through the channel $\mathbf{P}_k\mathcal{H}_k$ with the corresponding received noisy signal vector being $\mathbf{z}_k$ and the resulting decision metric is calculated from (16). The minimization in (16) requires the computation of M metrics since $\mathbf{x}$ is drawn from a rotated M-QAM constellation. Therefore, by using (16), we decompose the system into symbols and we decode each symbol independently from the others and as a result, we reduce the total decoding complexity in (3) from M^{2K} to $2KM$ which corresponds to a linear decoding complexity. According to our design structure in (5), the equivalent channel matrix for the new STBCs has the following general form

$$\mathcal{H}=\begin{bmatrix}\left(\mathcal{H}^1\right)^T & \left(\mathcal{H}^2\right)^T & \ldots & \left(\mathcal{H}^{n_R}\right)^T\end{bmatrix}^T \in \mathbb{R}^{3n_Tn_R\times 4K} \quad (17)$$

where

$$\mathcal{H}^l_{(3n_T\times 4K)}=\begin{bmatrix}\mathcal{H}^l_{\text{CIOD}} & \mathbf{0}_{(n_T\times 2K)} \\ \mathbf{0}_{(n_T\times 2K)} & \mathcal{H}^l_{\text{CIOD}}\end{bmatrix} \text{ for } l=1,2,\ldots,n_R$$

and $\mathcal{H}^l_{\text{CIOD}} \in \mathbb{R}^{2n_T\times 2K}$ is the equivalent channel matrix of the corresponding CIOD used for the construction of the new STBC.

IV. Full-Diversity Criteria for Symbol-Wise Decoding of New STBCs

In this section, we prove that the proposed STBCs can achieve full-diversity with the linear receivers presented in the previous section for any optimally rotated square M-QAM constellation. We give the two criteria for our STBC to achieve full-diversity with PIC based decoder given in (16):

- The proposed STBC must achieve full-diversity with ML receiver, i.e., the codeword difference matrix $\mathbf{X}-\hat{\mathbf{X}}$ must be full-rank, for all pairs $\mathbf{X},\hat{\mathbf{X}}$ with $\mathbf{X}\neq\hat{\mathbf{X}}$.
- $\mathcal{H}_0,\mathcal{H}_1,\ldots,\mathcal{H}_{2K-1}$ of (10) must create a linearly independent vector set containing $\mathbf{h}_{i,R}$ or $\mathbf{h}_{i,I}$ for $i=0,\ldots,2K-1$.

These criteria are an extension of those given in [15] where the complex transmission model is considered. In the following, we prove that the proposed STBC design guarantees these two criteria for any rotated square M-QAM constellation. To prove this fact, firstly we have to show that the minimum determinant $\delta_{\min}$ of the codeword distance matrix $(\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^H$ is non-zero for any codeword pairs

$\mathbf{X}$ and $\hat{\mathbf{X}}$ of the proposed STBCs with $\mathbf{X} \neq \hat{\mathbf{X}}$. Let $\Delta x_{iR} = x_{iR} - \hat{x}_{iR}$ and $\Delta x_{iI} = x_{iI} - \hat{x}_{iI}$ denote for $i = 0,1,\ldots,2K-1$ the differences in real and imaginary parts of the transmitted and erroneously detected information symbols x_i and $\hat{x}_i$, respectively, for any $x_i, \hat{x}_i \in \chi$ and $x_i \neq \hat{x}_i$. We calculate the $\delta_{\min}$ value of the new STBC for two transmit antennas as follows

$$\delta_{\min} = \min\Big\{\Delta x_{0R}^2\left(\Delta x_{0I}^2 + \Delta x_{1R}^2 + \Delta x_{3R}^2\right) + \Delta x_{1I}^2\left(\Delta x_{0I}^2 + \Delta x_{1R}^2 + \Delta x_{2I}^2\right) + \Delta x_{2I}^2\left(\Delta x_{0R}^2 + \Delta x_{2R}^2 + \Delta x_{3I}^2\right) + \Delta x_{3R}^2\left(\Delta x_{1I}^2 + \Delta x_{2R}^2 + \Delta x_{3I}^2\right)\Big\}. \tag{18}$$

It is obvious that (18) takes its minimum value when only one information symbol is erroneous (for example $x_0 \neq \hat{x}_0$) and the resulting minimum determinant is $\delta_{\min} = \min(\Delta x_{0R}^2 \Delta x_{0I}^2)$. Constellation rotation ensures a nonzero $\delta_{\min}$ value for any square M-QAM constellation. We choose the constellation rotation angle to maximize the $\delta_{\min}$ value for the proposed STBCs. The optimum rotation angle for square M-QAM with symbols having odd integer coordinates is found to be 31.72° which gives a $\delta_{\min}$ value of 3.2. With a similar analysis, we obtain the $\delta_{\min}$ value of the new STBCs for three and four transmit antennas as equal to 2.7 and 10.24, respectively. Due to lack of space, details are omitted.

After we guarantee the full-rank property, to ensure full-diversity, we have to show that the proposed STBC structure satisfies the second criterion. The orthogonality holds between the first and the last $2K$ columns of $\mathcal{H}$ in (17) due to the orthogonality of the CIOD, i.e., $\mathcal{H}_i \perp \mathcal{H}_j$, $i,j = 0,1,\ldots,K-1$, $i \neq j$, $\mathcal{H}_k \perp \mathcal{H}_l$, $k,l = K, K+1,\ldots,2K-1$, $k \neq l$. Due to the zero submatrices in (5) (and as a result in (17)) the proposed design ensures the linear independence condition, namely, the two columns of $\mathcal{H}_i$, $i = 0,1,\ldots,K-1$ cannot be expressed as any linear combination of the last $2K$ columns of $\mathcal{H}$ together (one column of $\mathcal{H}_i$ can be expressed while the other column cannot), and vice versa. Therefore, for a complex information symbol x_i, there exists a real vector ($\mathbf{h}_{i,R}$ or $\mathbf{h}_{i,I}$) such that cannot be expressed as any linear combination of the rest of the columns of $\mathcal{H}$. There are several STBC structures [7-8] that have higher symbol rates than the proposed STBCs, however, due to linear dependence between columns of their equivalent channel matrix $\mathcal{H}$, suboptimum decoding techniques (such as PIC group decoding) cannot guarantee full-diversity for them.

V. A Modified Decoding Algorithm for New STBCs

According to the PIC decoder given in Section III, all symbols in (5) are decoded independently whatever their decoding order. Furthermore, due to the non-orthogonal structure of the proposed STBC, it is not possible to decode some of the symbols using ML decoding. However, with the aid of SIC, we can perform ML decoding for the half of the symbols due to the special structure of the proposed STBC design. Consider the decoding problem of the STBC in (5). The proposed PIC-SIC-ML decoding algorithm is then as follows,

1) Decode the first K symbols using PIC group decoding algorithm (16) and obtain $\hat{\mathbf{x}}_i^{PIC}$ (or equivalently x_i^{PIC}) for $i = 0,1,\ldots,K-1$.
2) Under the assumption $\hat{\mathbf{x}}_i^{PIC} = \mathbf{x}_i$ for $i = 0,1,\ldots,K-1$ remove the interferences of the first K symbols from the received signal such as
$$\mathbf{y}' = \mathbf{y} - \sum_{i=0}^{K-1} \mathcal{H}_i \hat{\mathbf{x}}_i^{PIC}. \tag{19}$$
3) After the SIC operation, the receiver obtains the following channel output,
$$\mathbf{y}' = \mathcal{H}\overline{\mathbf{x}} + \mathbf{n} \tag{20}$$

where $\overline{\mathbf{x}} = [x_{KR} \quad x_{KI} \quad \ldots \quad x_{(2K-1)R} \quad x_{(2K-1)I}]^T$ and $\mathcal{H} = [(\mathcal{H}^1)^T \quad (\mathcal{H}^2)^T \quad \ldots \quad (\mathcal{H}^{n_R})^T]^T \in \mathbb{R}^{3T\times 2K}$ is the equivalent channel matrix where

$$\mathcal{H}^l = \begin{bmatrix} \mathbf{0}_{(T\times 2K)} \\ \mathcal{H}_{\text{CIOD}}^l \end{bmatrix} \tag{21}$$

with $\mathcal{H}_{\text{CIOD}}^l \in \mathbb{R}^{2T\times 2K}$ being the equivalent channel matrix of the corresponding CIOD. Since the columns of $\mathcal{H}^l$ in (21) are orthogonal to each other, we can decode the last K symbols with ML decoding via easy decomposition [15] as $x_i^{ML} = \arg\min_{\mathbf{x}\in\chi} \left\|\mathbf{y}' - \mathcal{H}_i \mathbf{x}\right\|$ for $i = K, K+1,\ldots,2K-1$. It should be noted that the decoding complexities of the PIC decoder and the PIC-SIC-ML decoder are the same ($2KM$).

VI. Simulation Results

In this section, we present some simulation results for the proposed STBCs and make comparisons with the existing STBCs in the literature. In Fig. 1, we compare the bit error rate (BER) performances of the proposed STBCs given in (7) and (8) for 4-QAM and 3 receive antennas with respect to received signal-to-noise ratio (SNR). As seen from Fig.1, PIC-SIC-ML decoder provides approximately 0.5dB SNR advantage compared to the PIC decoder. In Fig. 2, we compare the BER performances of the proposed STBC of (7), Guo-Xia STBC [15], the Golden code [8] and Alamouti's STBC [1] for two transmit and three receive antennas. To obtain a spectral efficiency of 8 bits/sec/Hz, our code and the Guo-Xia STBC use 64-QAM while Golden code and Alamouti's STBC use 16-QAM and 256-QAM, respectively. For the linear decoding of our STBC, we use the PIC-SIC-ML decoder given in Section V, and for a fair comparison we also employ symbol-wise PIC decoders for Guo-Xia STBC and the Golden code. As seen from Fig. 2, in case of linear receivers, our new STBC achieves best error performance and Guo-Xia STBC and the Golden code do not achieve full-diversity. In Fig. 3, we compare the BER performance of our new STBC of (8) with those of the Guo-Xia STBC [15] and the CIOD for four transmit and three receive antennas. Our new STBC and Guo-Xia STBC use 64-QAM while the CIOD uses 256-QAM to obtain a spectral efficiency of 8 bits/sec/Hz. Similar to two transmit antennas case, linear receivers are used for all schemes. As seen from Fig. 3, unlike our new STBC and the CIOD, Guo-Xia STBC does not achieve full-diversity in case of symbol-wise decoding. Simulation results show that at a BER value of 10^{-5}, our new code provides approximately 2.4 and 3.3 dB SNR advantages compared to Guo-Xia STBC and the CIOD, respectively.

VII. CONCLUSIONS

We have proposed three new rate-4/3 coordinate interleaved STBCs for two, three and four transmit antennas. By inspiring the PIC group decoding algorithm, we have developed a novel linear decoder for the proposed STBCs. We have proved that the proposed STBCs can achieve full-diversity with linear receiver for any rotated square M-QAM constellation, therefore, the upper bound in symbols rates of non-orthogonal STBCs achieving full-diversity with linear receivers, which is one symbols pcu, is exceeded by the new schemes using coordinate interleaving.

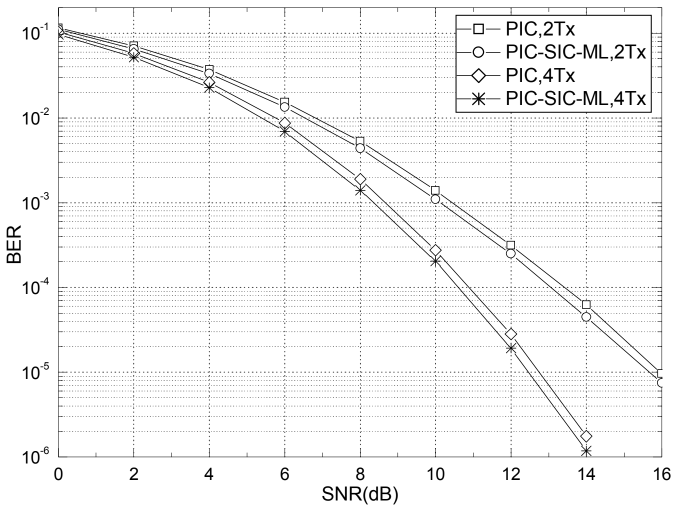

Fig.1: BER comparisons for PIC and PIC-SIC-ML decoders (4-QAM, 3Rx)

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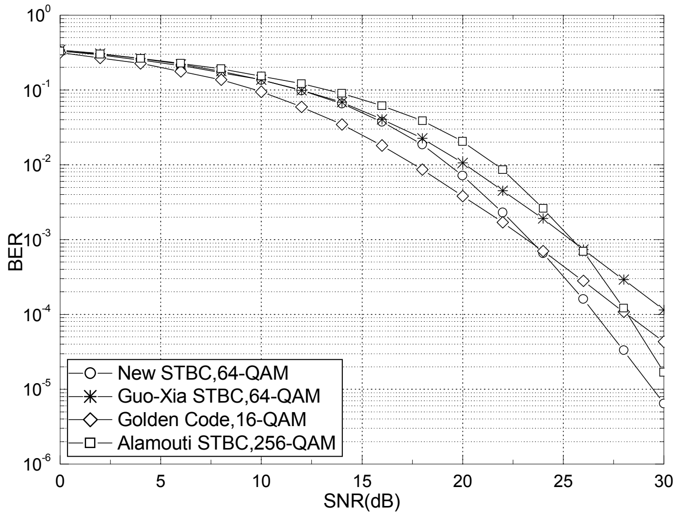

Fig.2 : BER comparisons for different STBCs at 8 bits/sec/Hz (2Tx & 3Rx)

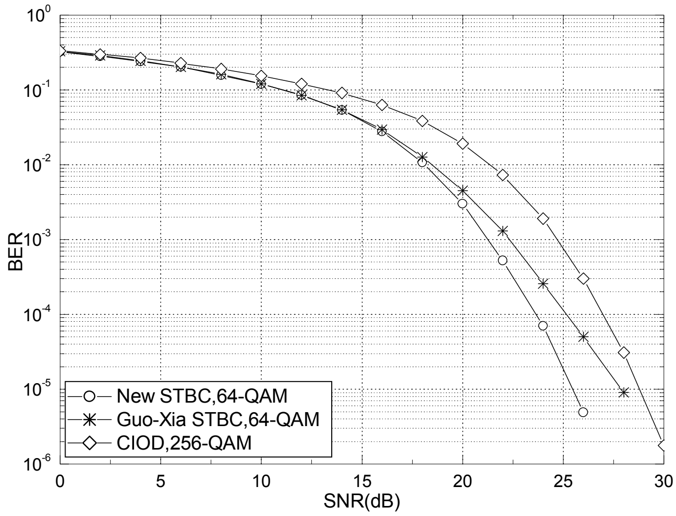

Fig.3 : BER comparisons for different STBCs at 8 bits/sec/Hz (4Tx & 3Rx)