This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE ICC 2011 proceedings

Trellis Code Design for Spatial Modulation

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Abstract*—In this paper, we propose a novel multiple-input multiple-output (MIMO) transmission scheme, called *trellis coded spatial modulation* (TC-SM) in which a trellis (convolutional) encoder and a spatial modulation (SM) mapper are jointly designed similar to the conventional trellis coded modulation (TCM). A soft decision Viterbi decoder, which is fed with the soft information supplied by the optimal SM decoder, is used at the receiver. The pairwise error probability (PEP) upper bound is derived for the TC-SM scheme in uncorrelated quasi-static Rayleigh fading channels. From the PEP upper bound, code design criteria are given and then used to obtain new 4-, 8- and 16-state TC-SM schemes using QPSK (quadrature phase-shift keying) and 8-PSK modulations for 2 and 3 bits/s/Hz spectral efficiencies. It is shown via computer simulations that the proposed TC-SM schemes achieve significantly better error performance than their counterparts at the same spectral efficiency, yet with reduced decoding complexity.

I. Introduction

A novel MIMO transmission scheme known as spatial modulation (SM) has been introduced in [1] as an alternative to the V-BLAST. The basic principle of SM is to use the indices of multiple antennas to convey information in addition to the conventional two dimensional signal constellations such as *M*-ary phase shift keying (*M*-PSK) and *M*-ary quadrature amplitude modulation (*M*-QAM), where *M* is the constellation size. Therefore, the information is conveyed by not only the amplitude/phase modulation techniques, but also by the antenna indices. Since only one transmit antenna is active during each symbol transmission, ICI is completely eliminated in SM and this results in much lower (linear) decoding complexity. Furthermore, SM does not require synchronization between the transmit antennas of the MIMO link and only one radio frequency (RF) chain is needed at the transmitter.

A trellis coded spatial modulation scheme has been proposed in [2,3], where the key idea of the trellis coded modulation (TCM) [4] is partially applied to SM to improve its performance in correlated channels. It has been shown in [2] that this scheme does not provide any error performance advantage compared to uncoded SM in uncorrelated channel conditions; on the other hand, the scheme of [2] does exhibit improved performance in correlated channels. The reason for this behavior can be explained by the trellis coding gain which does not have an impact on the performance when all the channel paths are uncorrelated. Here, we propose a different design method to construct a trellis coded SM scheme which benefits from the coding gain in uncorrelated channels.

In this paper, a novel MIMO transmission scheme, called *trellis coded spatial modulation* (TC-SM), which directly combines trellis coding and SM, is proposed. Similarly to conventional TCM, the trellis encoder and the SM mapper are jointly designed and a soft decision Viterbi decoder which is fed with the soft information supplied by the optimal SM decoder [5], is used at the receiver. The TC-SM mechanism, which switches between transmit antennas of a MIMO link, provides a type of virtual interleaving and offers an additional diversity gain, known as time diversity [6]. First, we derive the general conditional pairwise error probability (CPEP) upper bound of TC-SM and then, for quasi-static Rayleigh fading channels, by averaging over channel coefficients, we obtain the unconditional PEP (UPEP) of TC-SM for error events with path lengths two and three. Code design criteria are given for the TC-SM scheme, which are then used to obtain the best codes with optimized distance spectra. New TC-SM schemes with 4, 8 and 16 states are proposed for 2 and 3 bits/s/Hz spectral efficiencies. It is shown via computer simulations that the proposed TC-SM schemes for uncorrelated Rayleigh fading channels provide significant error performance improvements over space-time trellis codes (STTCs) [7] and the scheme given in [2] with a lower decoding complexity.

The organization of the paper is as follows. In Section II, we give our system model and introduce the new TC-SM scheme. In Section III, PEP upper bound for the TC-SM scheme is derived. Design criteria and design examples for TC-SM scheme are presented in Section IV. Simulation results and performance comparisons are given in Section V. Finally, Section VI includes the main conclusions of the paper.[§]

II. System Model

The considered TC-SM system model is given in Fig. 1. The independent and identically distributed (i.i.d.) binary

[§]*Notation*: Bold, lowercase and capital letters are used for column vectors and matrices, respectively. $(.)^*$, $(.)^T$ and $(.)^H$ denote complex conjugation, transposition and Hermitian transposition, respectively. $\mathbf{A}(p,q)$ represents the entry on the pth row and qth column of $\mathbf{A}$. $\det(\mathbf{A})$ and $\operatorname{rank}(\mathbf{A})$ denote the determinant and rank of $\mathbf{A}$, respectively. For a complex variable x, $\Re\{x\}$ denote the real part of x. The probability of an event is denoted by $\Pr(.)$. Probability distribution function (p.d.f.) of a random variable (r.v.) X is denoted by $f(x)$. $X \sim \mathcal{N}(m_X, \sigma_X^2)$ denotes the Gaussian distribution of a real r.v. X with mean m_X and variance σ_X^2. $X \sim \mathcal{CN}(0, \sigma_X^2)$ represents the distribution of a circularly symmetric complex Gaussian r.v. X. The number of elements in a set η is denoted as $n(\eta)$. χ represents a complex signal constellation of size M.

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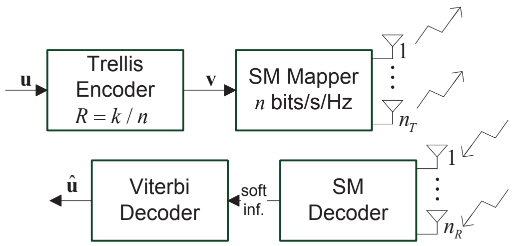

Fig. 1. TC-SM System Model

information sequence $\mathbf{u}$ is encoded by a rate $R = k/n$ trellis (convolutional) encoder whose output sequence $\mathbf{v}$ enters the SM mapper. The spatial modulator is designed in conjunction with the trellis encoder to transmit n coded bits in a transmission interval by means of the symbols selected from an M-level signal constellation such as M-PSK, M-QAM, etc., and of the antenna selected from a set of n_T transmit antennas such that $n = \log_2 (Mn_T)$. The SM mapper first specifies the identity of the transmit antenna determined by the first $\log_2 n_T$ bits of the coded sequence $\mathbf{v}$. It than maps the remaining $\log_2 M$ bits of the coded sequence onto the signal constellation employed for transmisson of the data symbols. Due to trellis coding, the overall spectral efficiency of the TC-SM would be k bits/s/Hz. The new signal generated by the SM is denoted by $x = (i, s)$ where $s \in \chi$ is the data symbol transmitted over the antenna labeled by $i \in \{1, 2, \cdots, n_T\}$. That is, the spatial modulator generates an $1 \times n_T$ signal vector $\begin{bmatrix} 0 \; 0 \cdots s \; 0 \cdots 0 \end{bmatrix}$ whose ith entry is s at the output of the n_T transmit antennas for transmission. The MIMO channel over which the spatially modulated symbols are transmitted, is characterized by an $n_T \times n_R$ matrix $\mathbf{H}$, whose entries are i.i.d. r.v.'s having the $\mathcal{CN}(0, 1)$ distribution, where n_R denotes the number of receive antennas. We assume that $\mathbf{H}$ remains constant during the transmission of a frame and takes independent values from one frame to another. We further assume that $\mathbf{H}$ is perfectly known at the receiver, but is not known at the transmitter. The transmitted signal is corrupted by an n_R-dimensional additive complex Gaussian noise vector with i.i.d. entries distributed as $\mathcal{CN}(0, N_0)$. At the receiver, a soft decision Viterbi decoder, which is fed with the soft information supplied by the optimal SM decoder, is employed to provide an estimate $\hat{\mathbf{u}}$ of the input bit sequence.

Let us introduce the concept of TC-SM by an example for $k = 2$ bits/s/Hz with $n_T = 4$. Consider an $R = 2/4$ trellis encoder with the generator matrix $\left[\begin{smallmatrix} 0 & 1+D & 0 & D \\ D & 0 & 1 & 0 \end{smallmatrix}\right]$, followed by the SM mapper. At each coding step, the first two coded bits determine the active transmit antenna over which the QPSK symbol determined by the last two coded bits is transmitted. The corresponding trellis diagram is depicted in Fig. 2, where each branch is labeled by the corresponding output bits and SM symbol (i, s), where $i \in \{1, 2, 3, 4\}$ and $s \in \{0, 1, 2, 3\}$.

This scheme differs from that of [2], in three basic ways. Firstly, to provide coding as well as diversity gain, all information bits are convolutionally encoded unlike in [2], in which only the information bits determining the corresponding antenna index are encoded. Thus, our joint encoding not

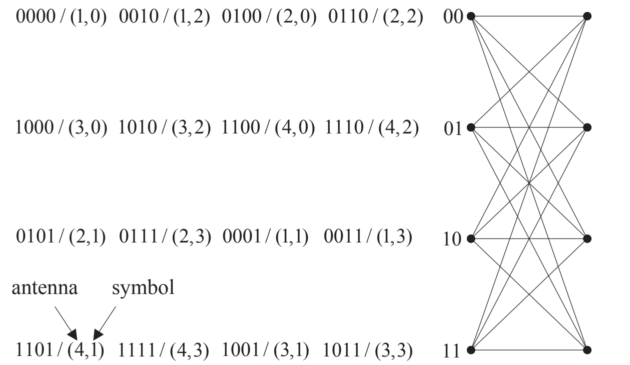

Fig. 2. Trellis diagram of the TC-SM scheme with $R = 2/4$ trellis encoder, four transmit antennas and QPSK, $k = 2$ bits/s/Hz.

only allows the operation of an optimum soft decoder at the receiver, and consequently improves the error performance of our system significantly. Secondly, an interleaver is not included in our scheme; however, we benefit from the TC-SM mechanism which acts as a virtual interleaver by switching between transmit antennas of a MIMO link to provide additional time diversity. Finally, a soft decision Viterbi decoder is employed at the receiver opposite to the hard decision Viterbi decoder of [2]. From these major differences in the operation of two schemes, we conclude that our scheme can be considered as being more directly inspired by Ungerboeck's TCM, in which the conventional M-PSK or M-QAM mapper of TCM is replaced by an SM mapper.

III. Pairwise-Error Probability (PEP) Derivation of the TC-SM Scheme

In this section, first, the conditional PEP (CPEP) of the TC-SM scheme is derived, and then for quasi-static Rayleigh fading channels, by averaging over channel fading coefficients, the unconditional PEP (UPEP) of the TC-SM scheme is obtained for error events with path lengths two and three. For the sake of simplicity, one receive antenna is assumed; however, all results can be easily extended to any number of receive antennas. Let $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ be a sequence of spatially modulated symbols to be transmitted, where $x_n = (i_n, s_n)$ is related to s_n, which is the symbol transmitted from i_nth antenna $(1 \leq i_n \leq n_T)$ at the nth transmission interval. Then the received signal is given as $y_n = \alpha_n s_n + w_n$, for $1 \leq n \leq N$, where α_n is the complex fading coefficient from i_nth transmit antenna to the receiver at the nth transmission inverval, and w_n is the noise sample with $\mathcal{CN}(0, N_0)$. A pairwise error event of length N occurs when the Viterbi decoder decides in favor of $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N)$ when $\mathbf{x}$ is transmitted. Let $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_N)$ and $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_N)$ denote the sequences of fading coefficients corresponding to transmitted and erroneously detected SM symbol sequences, $\mathbf{x}$ and $\hat{\mathbf{x}}$, respectively. The CPEP for this case is given by

$$\Pr\left(\mathbf{x} \to \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) = \Pr\left\{ m\left(\mathbf{y}, \hat{\mathbf{x}}; \boldsymbol{\beta}\right) \geq m\left(\mathbf{y}, \mathbf{x}; \boldsymbol{\alpha}\right) \middle| \mathbf{x} \right\} \quad (1)$$

where $m(\mathbf{y}, \mathbf{x}; \boldsymbol{\alpha}) = \sum_{n=1}^{N} m(y_n, s_n; \alpha_n) = -\sum_{n=1}^{N} |y_n - \alpha_n s_n|^2$ is the decision metric for $\mathbf{x}$, since

y_n is Gaussian when conditioned on α_n and s_n. Then, with simple manipulation, (1) can be expressed as

$$\begin{aligned} &Pr\left(\mathbf{x}\to\hat{\mathbf{x}}|\,\boldsymbol{\alpha},\boldsymbol{\beta}\right)\\ &=\Pr\left\{\sum_{n=1}^{N}|y_n-\alpha_n s_n|^2\geq\sum_{n=1}^{N}|y_n-\beta_n\hat{s}_n|^2\middle|\,\mathbf{x}\right\}\\ &=\Pr\left\{\sum_{n=1}^{N}-|\alpha_n s_n-\beta_n\hat{s}_n|^2+2\Re\{\tilde{w}_n\}\geq 0\middle|\,\mathbf{x}\right\}\end{aligned} \tag{2}$$

where $\tilde{w}_n = w_n(\beta_n^*\hat{s}_n^* - \alpha_n^* s_n^*)$. Denoting the nth term of the sum in (2) by d_n, we obtain $\Pr(\mathbf{x}\to\hat{\mathbf{x}}|\,\boldsymbol{\alpha},\boldsymbol{\beta}) = \Pr\{d\geq 0|\,\mathbf{x}\}$ where $d=\sum_{n=1}^{N} d_n$ is the decision variable to be compared with the zero threshold. Since, $\tilde{w}_n$ is Gaussian with $\mathcal{CN}\left(0, N_0|\beta_n^*\hat{s}_n^* - \alpha_n^* s_n^*|^2\right)$, it is straightforward to show that d is also Gaussian with $\mathcal{N}\left(m_d,\sigma_d^2\right)$ where, $m_d=-\sum_{n=1}^{N}|\alpha_n s_n-\beta_n\hat{s}_n|^2$ and $\sigma_d^2 = 2N_0\sum_{n=1}^{N}|\alpha_n s_n-\beta_n\hat{s}_n|^2$. Finally, the CPEP of the TC-SM scheme is calculated from (2) as

$$\Pr\left(\mathbf{x}\to\hat{\mathbf{x}}|\,\boldsymbol{\alpha},\boldsymbol{\beta}\right) = Q\left(\frac{-m_d}{\sigma_d}\right) = Q\left(\sqrt{\frac{\sum_{n=1}^{N}A_n}{2N_0}}\right) \tag{3}$$

where $A_n = |\alpha_n s_n - \beta_n\hat{s}_n|^2$. Using the bound $Q(x)\leq\frac{1}{2}e^{-x^2/2}$, the CPEP of the TC-SM scheme can be upper bounded by*

$$\Pr\left(\mathbf{x}\to\hat{\mathbf{x}}|\,\boldsymbol{\alpha},\boldsymbol{\beta}\right)\leq\frac{1}{2}\exp\left(-\frac{\gamma}{4}\sum\nolimits_{n=1}^{N}|\alpha_n s_n-\beta_n\hat{s}_n|^2\right) \tag{4}$$

where $\gamma = E_s/N_0 = 1/N_0$ is the average received signal-to-noise ratio (SNR). The CPEP upper bound of the TC-SM scheme, which is given in (4), can be alternatively rewritten in matrix form as

$$\Pr\left(\mathbf{x}\to\hat{\mathbf{x}}|\,\boldsymbol{\alpha},\boldsymbol{\beta}\right)\leq\frac{1}{2}\exp\left(-\frac{\gamma}{4}\mathbf{h}^H\mathbf{S}\mathbf{h}\right) \tag{5}$$

where $\mathbf{h}=\left[h_1\ h_2\cdots h_{n_T}\right]^T$ is the $n_T\times 1$ channel vector with $h_i, i=1,2,\cdots,n_T$ representing the channel fading coefficient from ith transmit antenna to the receiver, which is assumed to be constant through the error event. $\mathbf{S}=\sum_{n=1}^{N}\mathbf{S}_n$ where $\mathbf{S}_n$ is an $n_T\times n_T$ Hermitian matrix representing a realization of α_n and β_n which are related to the channel coefficients as $\alpha_n = h_{i_n}$, $\beta_n = h_{j_n}$, i_n and $j_n\in\{1,2,\cdots,n_T\}$ being the transmitted and detected antenna indices, respectively. The entries of the matrix $\mathbf{S}_n$, $n=1,2,\cdots,N$ are given as follows:

For $i_n\neq j_n$

$$\mathbf{S}_n(p,q)=\begin{cases}|s_n|^2, & \text{if } p=q=i_n\\ |\hat{s}_n|^2, & \text{if } p=q=j_n\\ -s_n^*\hat{s}_n, & \text{if } p=i_n, q=j_n\\ -s_n\hat{s}_n^*, & \text{if } p=j_n, q=i_n\\ 0, & \text{otherwise}\end{cases} \tag{6}$$

*Note that, if $\alpha_n=\beta_n$ for all n, $1\leq n\leq N$, (4) reduces to the CPEP upper bound of TCM [6].

and for $i_n = j_n$

$$\mathbf{S}_n(p,q)=\begin{cases}d_{E_n}^2, & \text{if } p=q=i_n\\ 0, & \text{otherwise}\end{cases} \tag{7}$$

where $d_{E_n}^2 = |s_n-\hat{s}_n|^2$. As an example, for $n_T=4$ with $\alpha_n=h_1$ and $\beta_n=h_3$ (i.e., $i_n=1$ and $j_n=3$) $\mathbf{S}_n$ is obtained as

$$\mathbf{S}_n=\begin{bmatrix}|s_n|^2 & 0 & -s_n^*\hat{s}_n & 0\\ 0&0&0&0\\ -s_n\hat{s}_n^* & 0 & |\hat{s}_n|^2 & 0\\ 0&0&0&0\end{bmatrix}. \tag{8}$$

In order to obtain the UPEP of the TC-SM scheme, (5) should be averaged over the multivariate complex Gaussian p.d.f. of $\mathbf{h}$ which is given as $f(\mathbf{h}) = (1/\pi^{n_T})\,e^{-\mathbf{h}^H\mathbf{h}}$ since the entries of $\mathbf{h}$ are i.i.d. with p.d.f. $\mathcal{CN}(0,1)$. UPEP upper bound of the TC-SM is calculated from (5) as

$$\begin{aligned}\Pr\left(\mathbf{x}\to\hat{\mathbf{x}}\right)&\leq\frac{1}{2}\int_{\mathbf{h}}\pi^{-n_T}\exp\left(-\frac{\gamma}{4}\mathbf{h}^H\mathbf{S}\mathbf{h}\right)\exp\left(-\mathbf{h}^H\mathbf{h}\right)d\mathbf{h}\\ &=\frac{1}{2}\int_{\mathbf{h}}\pi^{-n_T}\exp\left(-\mathbf{h}^H\boldsymbol{\Sigma}^{-1}\mathbf{h}\right)d\mathbf{h}\end{aligned} \tag{9}$$

where $\boldsymbol{\Sigma}^{-1}=\left[\frac{\gamma}{4}\mathbf{S}+\mathbf{I}\right]$ and $\mathbf{I}$ is the $n_T\times n_T$ identity matrix. Since $\boldsymbol{\Sigma}$ is a Hermitian and positive definite complex covariance matrix, the integrand of the multivariate complex Gaussian p.d.f given in (9) yields the following result:

$$\Pr\left(\mathbf{x}\to\hat{\mathbf{x}}\right)\leq\frac{1}{2}\det\left(\boldsymbol{\Sigma}\right)=\frac{1}{2\det\left(\frac{\gamma}{4}\mathbf{S}+\mathbf{I}\right)}. \tag{10}$$

With simple manipulation, (10) can be expressed as $\Pr(\mathbf{x}\to\hat{\mathbf{x}})\leq\left(2\left(\frac{\gamma}{4}\right)^b\prod_{i=1}^{b}\lambda_i^{\mathbf{S}}\right)^{-1}$, where $\lambda_i^{\mathbf{S}}$ is the ith eigenvalue of $\mathbf{S}$ and $b=\text{rank}(\mathbf{S})$. Although (10) gives an effective and simple way to evaluate the UPEP upper bound of TC-SM scheme in closed form, for an error event with path length N, the matrix $\mathbf{S}$ has $(n_T)^{2N}$ possible realizations which correspond to all of the possible transmitted and detected antenna indices along this error event. However, due to the special structure of $\mathbf{S}$, these $(n_T)^{2N}$ possible realizations can be grouped into a small number of distinct types having the same UPEP upper bound, and the resulting upper bound calculated from (10) is mainly determined by the number of degrees of freedom (DOF) of the error event which is defined as follows:

Definition 1: For an error event with path length N, the number of degrees of freedom (DOF) is defined as the total number of different channel fading coefficients in $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ sequences. It can be easily shown that DOF $\leq 2N$. For example, for $N=2$, $\boldsymbol{\alpha}=(\alpha_1,\alpha_2)$ and $\boldsymbol{\beta}=(\beta_1,\beta_2)$, DOF $=3$ if $\alpha_1=\beta_1\neq\alpha_2\neq\beta_2$. Besides the DOF, a second fact, which is explained as follows, determines the result of (10). Let us rewrite (4) as

$$\begin{aligned}\Pr\left(\mathbf{x}\to\hat{\mathbf{x}}|\,\boldsymbol{\alpha},\boldsymbol{\beta}\right)\leq\frac{1}{2}\exp\Bigg(-\frac{\gamma}{4}\Big[&\sum\nolimits_{\eta}|\alpha_n|^2|s_n-\hat{s}_n|^2\\ &+\sum\nolimits_{\tilde{\eta}}|\alpha_n s_n-\beta_n\hat{s}_n|^2\Big]\Bigg)\end{aligned} \tag{11}$$

TABLE I
UPEP VALUES FOR ERROR EVENTS WITH PATH LENGTH TWO

Case	PEP(high SNR)
$n(\eta)=0, \text{DOF}=2^*$	$4/(1-\cos\theta)\gamma^2$
$n(\eta)=0, \text{DOF}=3$	$8/3\gamma^2$
$n(\eta)=0, \text{DOF}=3$	$2/\gamma^2$
$n(\eta)=1, \text{DOF}=2$	$8/d_{E_m}^2\gamma^2$
$n(\eta)=1, \text{DOF}=3$	$4/d_{E_m}^2\gamma^2$

TABLE II
UPEP VALUES FOR ERROR EVENTS WITH PATH LENGTH THREE

Case	PEP(high SNR)
$n(\eta)=0, \text{DOF}=3$	$16/\left(1-\cos\tilde{\theta}\right)\gamma^3$
$n(\eta)=0, \text{DOF}=3^*$	$16/(1-\cos\theta)\gamma^3$
$n(\eta)=0, \text{DOF}=4$	$8/\gamma^3$
$n(\eta)=0, \text{DOF}=4^*$	$8/(1-\cos\theta)\gamma^3$
$n(\eta)=0, \text{DOF}=5$	$16/3\gamma^3$
$n(\eta)=0, \text{DOF}=6$	$4/\gamma^3$
$n(\eta)=1, \text{DOF}=2^*$	$4/\left(1+d_{E_m}^2-\cos\theta\right)\gamma^2$
$n(\eta)=1, \text{DOF}=3$	$32/d_{E_m}^2\gamma^3$
$n(\eta)=1, \text{DOF}=3^*$	$16/(1-\cos\theta)d_{E_m}^2\gamma^3$
$n(\eta)=1, \text{DOF}=4$	$32/3d_{E_m}^2\gamma^3$
$n(\eta)=1, \text{DOF}=5$	$8/d_{E_m}^2\gamma^3$

where η and $\tilde{\eta}$ are the sets of all n for which $\alpha_n=\beta_n$ and $\alpha_n\neq\beta_n$, respectively, and $n(\eta)+n(\tilde{\eta})=N$. The first term in (11) corresponds to the TCM term while the second term corresponds to the SM term. Note that in some cases, the same DOF value can be supported with different $n(\eta)$ and $n(\tilde{\eta})$ values, and this also affects the result of (10).

In Tables I and II, for the aforementioned different cases, the resulting UPEP upper bounds at high SNR values ($\gamma\gg 1$) are given for error events with path lengths $N=2$ and 3, respectively, where a constant envelope M-PSK constellation is assumed and $\theta=\pm\Delta\theta_1\pm\Delta\theta_2$, $\tilde{\theta}=\pm\Delta\theta_1\pm\Delta\theta_2\pm\Delta\theta_3$, $\Delta\theta_n=\theta_n-\hat{\theta}_n, n=1,2,3$ and $s_i=e^{j\theta_i}, \hat{s}_i=e^{j\hat{\theta}_i}$ with $\theta_i,\hat{\theta}_i\in\left\{\frac{2\pi r}{M}, r=0,\cdots,M-1\right\}$ and $m\in[1,N]$. The asterisk for DOF values means the considered UPEP value is dependent on θ. As seen from these tables, for an error event with path length N, a diversity order of N is achieved if DOF $\geq N$. The following theorem generalizes this fact. Due to space limitations, the proof is omitted.

Theorem 1: In case of an error event with path length N, in order to achieve a diversity order of N (an UPEP upper bound of a/γ^N for $\gamma\gg 1$ and $a\in\mathbb{R}^+$), a necessary condition is DOF $\geq N$.

This theorem, which can be proved by showing that the rank of the matrix $\mathbf{S}$ is equal to N (i.e. $b=N$) if DOF $\geq N$, constitutes the basis of our TC-SM design criteria.

IV. TC-SM CODE DESIGN CRITERIA AND DESIGN EXAMPLES

In this section, we give design criteria for TC-SM scheme and provide some code design examples for spectral efficiencies $k=2$ and 3 bits/s/Hz. By considering the UPEP

TABLE III
GENERATOR MATRICES OF DIFFERENT STATE TC-SM SCHEMES FOR 2 AND 3 BITS/S/HZ

State	$k=2$ bits/s/Hz	$k=3$ bits/s/Hz
4	$\begin{bmatrix} 0 & 1+D & 0 & D \\ D & 0 & 1 & 0 \end{bmatrix}$	-
8	$\begin{bmatrix} 0 & D & 1 & D \\ D+D^2 & 1 & 0 & D^2 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & D & 0 & D & 0 \\ 0 & D & 1 & 0 & 0 & D \\ D & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$
16	$\begin{bmatrix} 1+D^2 & D^2 & D+D^2 & 0 \\ D^2 & 1 & 0 & D+D^2 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & D & 0 & D & 0 \\ 0 & D & 0 & 1 & 0 & D \\ D+D^2 & 0 & 1+D^2 & 0 & D^2 & D^2 \end{bmatrix}$

analysis of the previous section, the following design criteria are derived for the TC-SM scheme:

1) *Diversity gain criterion*: For a trellis code with minimum error event length N, to achieve a diversity order of N, DOF must be greater than or equal to N for all error events with path length greater than or equal to N.
2) *Coding gain criterion*: After ensuring maximum diversity gain, the distance spectrum of the TC-SM should be optimized by considering the UPEP values calculated from (10).

In Table III, we give the generator matrices of the TC-SM codes for $k=2$ and 3 bits/s/Hz spectral efficiencies and different numbers of states. All codes are designed manually according to the TC-SM design criteria given above. For 2 bits/s/Hz transmission, we use four transmit antennas with a QPSK constellation, while for 3 bits/s/Hz, we use eight transmit antennas with an 8-PSK constellation since, $R=2/4$ and $R=3/6$ trellis encoders are employed to obtain $k=2$ and 3 bits/s/Hz, respectively. For 2 bits/s/Hz, we optimized the distance spectra of our 4- and 8-state codes to maximize the number of error events with DOF >2. On the other hand, our 16-state code is designed such that the error events with $N\geq 3$ ensure DOF ≥ 3, and therefore, diversity order of three is achieved. For 3 bits/s/Hz, in the same manner as performed previously, we optimized the distance spectra of 8- and 16-state codes, guaranteed DOF ≥ 2 and maximized the number of error events with DOF >2. In all of our TC-SM constructions, we assigned SM symbols to the branches of the trellis in such a way that a catastrophic code is avoided. We also guarantee DOF ≥ 2 for higher values of N to maintain the diversity order of the system.

V. SIMULATION RESULTS AND COMPARISONS

In this section, we present simulation results for the TC-SM scheme with different configurations and make comparisons with the reference systems. The bit error rate (BER) performance of these schemes was evaluated via Monte Carlo simulations for various spectral efficiencies and numbers of states as a function of the average SNR per receive antenna. In all cases, the decision depth of the Viterbi decoder was chosen to be 20, which corresponds to a frame length of $20k$ bits for both TC-SM and STTC schemes [7] at k bits/s/Hz.

First, we give simulation results for 2 bits/s/Hz transmission with one and two receive antenna cases. In Fig. 3, the BER performance of the proposed 4-,8- and 16-state TC-SM schemes is compared with 4-,8- and 16-state optimal QPSK STTCs with two transmit antennas. As seen from this figure, TC-SM schemes offer a significant improvement in BER performance compared to the STTCs. It is also observed that with increasing number of states, the performance gap between TC-SM and STTC schemes increases since TC-SM provides higher coding gains with increasing complexity. Note that our 16-state code offers a major improvement in BER performance over 16-state STTC due to its third order time diversity compared to the second order diversity of STTC.

In Fig. 4, for 3 bits/s/Hz transmission, the BER performance of the proposed 8- and 16-state TC-SM schemes is compared with 8- and 16-state optimal 8-PSK STTCs for two transmit antennas. Since we use eight transmit antennas, the error performance gap between TC-SM schemes and STTCs becomes higher than that of 2 bits/s/Hz case in favor of TC-SM since error events with higher lengths contributes UPEP values with higher diversity orders. For comparison, the BER performance of the scheme given in [2] is also shown in Fig. 4. To achieve the required spectral efficiency, this scheme uses four transmit antennas and QPSK and the octal generator sequence of the $R = 1/2$ trellis encoder employed was chosen as $[5, 2]$. We observe that our scheme offers significant improvement over the scheme given in [2] due to its second error transmit diversity and high coding gain advantage.

Since only one transmit antenna is active in our scheme contrary to the reference STTCs with the same trellis structure in which two antennas transmit simultaneously, for a single metric calculation of the STTC decoder, the required number of complex multiplications and complex additions are equal to three and two, while the corresponding values for our TC-SM codes are two and one, respectively. Therefore, for 2 bits/s/Hz, TC-SM provides 25% and 33% reductions in the number of real multiplications and real additions per single branch metric calculation of the Viterbi decoder, respectively, and these values being increased to 30% and 37.5% for 3 bits/s/Hz. From an implementation point of view, unlike the STTCs, our scheme requires only one RF chain at the transmitter, even if we have a higher number of transmit antennas, and requires no synchronization between them.

VI. Conclusions

In this paper, we have introduced a novel coded MIMO transmission scheme which directly combines trellis coding and SM. Although one transmit antenna is active for transmission, we benefit from the time diversity provided by the TC-SM mechanism, which creates a kind of virtual interleaving by switching between the transmit antennas of a MIMO link, for quasi-static fading channels. A total of five TC-SM codes have been proposed according to the TC-SM design criteria to obtain best performance. We have shown that the proposed schemes offer significant error performance improvements over STTCs with a lower decoding complexity for 2 and 3 bits/s/Hz transmissions. Our future work will be focused on to increasing the spectral efficiency of TC-SM scheme and analysis for correlated channels.

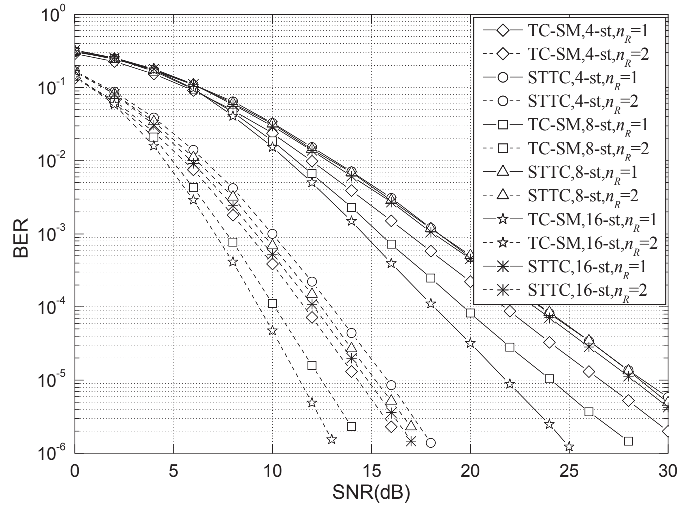

Fig. 3. BER performance for 4-,8- and 16-state TC-SM and STTC schemes at 2 bits/s/Hz

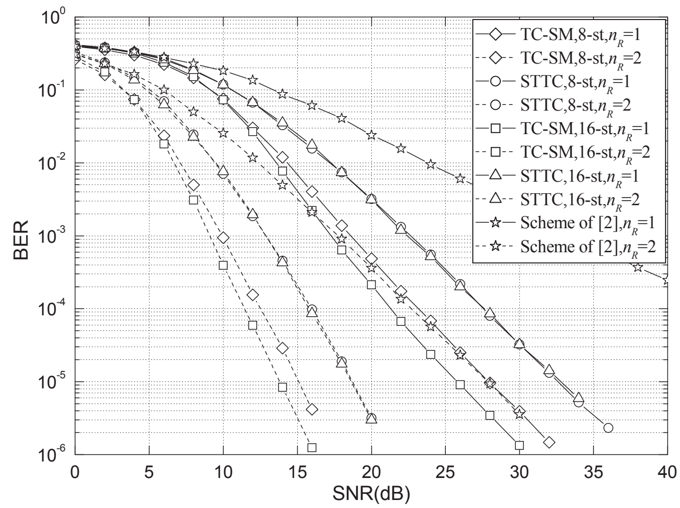

Fig. 4. BER performance for 8- and 16-state TC-SM and STTC schemes with the system of [2] at 3 bits/s/Hz

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