Orthogonal Frequency Division Multiplexing with Index Modulation

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***Abstract*—In this paper, a novel orthogonal frequency division multiplexing (OFDM) scheme, which is called OFDM with index modulation (OFDM-IM), is proposed for frequency-selective fading channels. In this scheme, inspiring from the recently introduced spatial modulation concept for multiple-input multiple-output (MIMO) channels, the information is conveyed not only by M-ary signal constellations as in classical OFDM, but also by the indices of the subcarriers, which are activated according to the incoming bit stream. Different transceiver structures are proposed and a theoretical error performance analysis is provided for the new scheme. It is shown via computer simulations that the proposed scheme achieves significantly better error performance than classical OFDM due to the information bits carried in the spatial domain by the indices of OFDM subcarriers.**

I. Introduction

Multicarrier transmission has become a key technology for wideband digital communications in recent years and has been included in many wireless standards. Orthogonal frequency division multiplexing (OFDM) has been the most popular multicarrier transmission technique in wireless communications. Similarly, multiple-input multiple-output (MIMO) transmission techniques have been implemented in many practical applications, due to their benefits over single antenna systems. Spatial modulation (SM), which uses the spatial domain to convey information in addition to the classical signal constellations, has emerged as a promising MIMO transmission technique [1], [2]. The application of the SM principle to the subcarriers of an OFDM system has been proposed in [3]. However, in this scheme, the number of active OFDM subcarriers varies for each OFDM block, and furthermore, a kind of perfect feedforward is assumed from the transmitter to the receiver via the excess subcarriers to explicitly signal the mapping method for the subcarrier index selecting bits.

In this paper, taking a different approach from [3], we propose a novel transmission scheme called *OFDM with index modulation (OFDM-IM)* for frequency selective fading channels. In this scheme, information is conveyed not only by M-ary signal constellations as in classical OFDM, but also by the indices of the subcarriers, which are activated according to the incoming information bits. Unlike the scheme of [3], feedforward signaling from transmitter to the receiver is not required in our scheme in order to successfully detect the transmitted information bits. A general method, by which the number of active subcarriers can be adjusted, and the incoming bits can be systematically mapped to these active subcarriers, is presented for the OFDM-IM scheme. Different mapping and detection techniques for the new scheme are proposed, and it is shown via computer simulations and also supported by a theoretical error performance analysis that the proposed scheme achieves significantly better bit error rate (BER) performance than the classical OFDM.

The rest of the paper can be summarized as follows. In Section II, the system model of OFDM-IM is presented. In Section III, we propose different implementation approaches for OFDM-IM. The theoretical error performance of OFDM-IM is investigated in Section IV. The computer simulation results are given in Section V. Finally, Section VI concludes the paper.

Notation: Bold, lowercase and capital letters are used for column vectors and matrices, respectively. $(\cdot)^T$ and $(\cdot)^H$ denote transposition and Hermitian transposition, respectively. $\det(\mathbf{A})$ and $\mathrm{rank}(\mathbf{A})$ denote the determinant and rank of $\mathbf{A}$, respectively. $\lambda_i(\mathbf{A})$ is the ith eigenvalue of $\mathbf{A}$, where $\lambda_1(\mathbf{A})$ is the largest eigenvalue. $\mathbf{I}_{N\times N}$ and $\mathbf{0}_{N_1\times N_2}$ are the identity and zero matrices with dimensions $N\times N$ and $N_1\times N_2$, respectively. $\|\cdot\|_F$ stands for the Frobenius norm. The probability of an event is denoted by $P(\cdot)$ and $E\{\cdot\}$ stands for expectation. The probability density function (p.d.f.) of a random vector $\mathbf{x}$ is denoted by $f(\mathbf{x})$. $X\sim\mathcal{CN}(0,\sigma_X^2)$ represents the distribution of a circularly symmetric complex Gaussian r.v. X with variance σ_X^2. $Q(\cdot)$ denotes the tail probability of the standard Gaussian distribution. $C(n,k)$ denotes the binomial coefficient and $\lfloor\cdot\rfloor$ is the floor function. $\mathcal{S}$ denotes the complex signal constellation of size M.

II. System Model of OFDM-IM

Let us consider an OFDM-IM scheme which is operating on a frequency-selective Rayleigh fading channel. A total of m information bits enter the OFDM-IM transmitter for the

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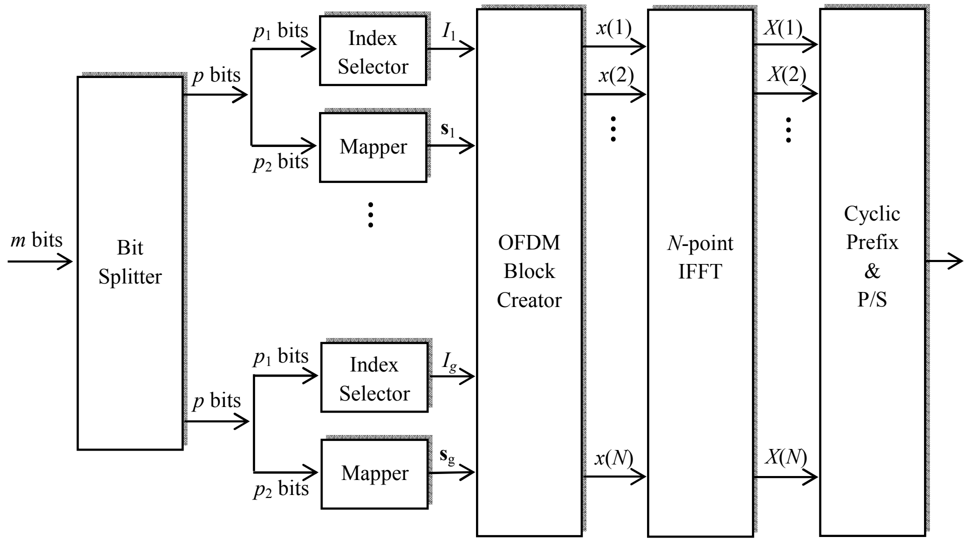

Fig. 1. Block Diagram of the OFDM-IM Transmitter

transmission of each OFDM block. These m bits are then split into g groups each containing p bits, i.e.,

$$m = pg. \tag{1}$$

Each group of p-bits is mapped to an *OFDM subblock* of length n, where $n = N/g$ and N is the number of OFDM subcarriers, i.e., the size of the fast Fourier transform (FFT). Contrary to the classical OFDM, this mapping operation is not only performed by means of the modulated symbols, but also by the indices of the subcarriers. Inspiring from the SM concept, additional information bits are transmitted by a subset of the OFDM subcarrier indices. For each subblock, only k out of n available indices are employed for this purpose and they are determined by a selection procedure (different selection procedures are presented in Sec. III) from a predefined set of active indices, based on the first p_1 bits of the incoming p-bits sequence. We set the symbols corresponding to the inactive subcarriers to zero, therefore, we do not transmit data with them. The remaining

$$p_2 = k \log_2 M \tag{2}$$

bits of this sequence are mapped on to the M-ary signal constellation to determine the data symbols that modulate the subcarriers having active indices, therefore, we have $p = p_1 + p_2$. In other words, in the OFDM-IM scheme, the information is conveyed by both of the M-ary constellation symbols and the indices of the subcarriers that are modulated by these constellation symbols. Due to the fact that we do not use all of the available subcarriers, we compensate for the loss in the total number of transmitted bits by transmitting additional bits in the spatial domain of the OFDM block.

The block diagram of the OFDM-IM transmitter is given in Fig. 1. For each subblock β, the incoming p_1 bits are transferred to the index selector, which chooses k active indices out of n available indices, where the selected indices are given by

$$I_\beta = \{i_{\beta,1}, \ldots, i_{\beta,k}\} \tag{3}$$

where $i_{\beta,\gamma} \in [1, \ldots, n]$ for $\beta = 1, \ldots, g$ and $\gamma = 1, \ldots, k$. Therefore, for the total number of information bits carried by the spatial position of the active indices in the OFDM block, we have

$$m_1 = p_1 g = \lfloor \log_2 (C(n,k)) \rfloor g. \tag{4}$$

In other words, I_β has $c = 2^{p_1}$ possible realizations. On the other hand, the total number of information bits carried by the M-ary signal constellation symbols are given by

$$m_2 = p_2 g = k (\log_2 (M)) g \tag{5}$$

since the total number of active subcarriers is $K = kg$ in our scheme. Consequently, a total of

$$m = m_1 + m_2 \tag{6}$$

bits are transmitted by a single block of the OFDM-IM scheme. The vector of the modulated symbols at the output of the M-ary mapper (modulator), which carries p_2 bits, is given by

$$\mathbf{s}_\beta = \left[s_\beta(1) \ldots s_\beta(k) \right] \tag{7}$$

where $s_\beta(\gamma) \in \mathcal{S}$, $\beta = 1, \ldots, g$, $\gamma = 1, \ldots, k$. We assume that $E\{\mathbf{s}_\beta \mathbf{s}_\beta^H\} = k$, i.e., the signal constellation is normalized to have unit average power. The OFDM block creator creates all of the subblocks by taking into account I_β and $\mathbf{s}_\beta$ for all β first and it then forms the $N \times 1$ main OFDM block

$$\mathbf{x}_F = \left[x(1)\, x(2) \cdots x(N) \right]^T \tag{8}$$

where $x(\alpha) \in \{0, \mathcal{S}\}, \alpha = 1, \ldots, N$, by concatenating these g subblocks. Unlike the classical OFDM, in our scheme $\mathbf{x}_F$ contains some zero terms whose positions carry information.

After this point, the same procedures as those of classical OFDM are applied. The OFDM block is processed by the inverse FFT (IFFT) algorithm:

$$\mathbf{x}_T = \frac{N}{\sqrt{K}}\text{IFFT}\left\{\mathbf{x}_F\right\} = \frac{1}{\sqrt{K}}\mathbf{W}_N^H\mathbf{x}_F \tag{9}$$

where $\mathbf{x}_T$ is the time domain OFDM block, $\mathbf{W}_N$ is the discrete Fourier transform (DFT) matrix with $\mathbf{W}_N^H\mathbf{W}_N = N\mathbf{I}_N$ and the term $N/\sqrt{K}$ is used for the normalization $E\left\{\mathbf{x}_T^H\mathbf{x}_T\right\} = N$ (at the receiver, the FFT demodulator employs a normalization factor of $\sqrt{K}/N$). At the output of the IFFT, a cyclic prefix (CP) of length L samples $\left[X\left(N-L+1\right)\cdots X\left(N-1\right)X\left(N\right)\right]^T$ is appended to the beginning of the OFDM block. After parallel to serial (P/S) and digital-to-analog conversion, the signal is sent through a frequency-selective Rayleigh fading channel which can be represented by the channel impulse response (CIR) coefficients

$$\mathbf{h}_T = \left[h_T\left(1\right)...\,h_T\left(\nu\right)\right]^T \tag{10}$$

where $h_T\left(\sigma\right), \sigma = 1,...,\nu$ are circularly symmetric complex Gaussian random variables with the $\mathcal{CN}\left(0,\frac{1}{\nu}\right)$ distribution. Assuming that the channel remains constant during the transmission of an OFDM block and the CP length L is larger than ν, the equivalent frequency domain input-output relationship of the OFDM scheme is given by

$$y_F\left(\alpha\right) = x\left(\alpha\right)h_F\left(\alpha\right) + w_F\left(\alpha\right), \quad \alpha = 1,\ldots,N \tag{11}$$

where $y_F\left(\alpha\right)$, $h_F\left(\alpha\right)$ and $w_F\left(\alpha\right)$ are the received signals, the channel fading coefficients and the noise samples in the frequency domain, whose vector presentations are given as $\mathbf{y}_F$, $\mathbf{h}_F$ and $\mathbf{w}_F$, respectively. The distributions of $h_F\left(\alpha\right)$ and $w_F\left(\alpha\right)$ are $\mathcal{CN}\left(0,1\right)$ and $\mathcal{CN}\left(0,N_{0,F}\right)$, respectively, where $N_{0,F}$ is the noise variance in the frequency domain, which is related by the noise variance in the time domain by

$$N_{0,F} = \left(K/N\right)N_{0,T}. \tag{12}$$

We define the signal-to-noise ratio (SNR) as

$$\rho = E_b/N_{0,T} \tag{13}$$

where $E_b = \left(N+L\right)/m$ is the average transmitted energy per bit. The spectral efficiency of the OFDM-IM scheme is given by

$$m/\left(N+L\right) \text{ [bits/s/Hz].} \tag{14}$$

We assume that perfect channel state information is available at the receiver.

The receiver's task is to detect the indices of the active subcarriers and the corresponding information symbols by processing $y_F\left(\alpha\right), \alpha = 1,\ldots,N$. Unlike the classical OFDM, a simple maximum likelihood (ML) decision on $x\left(\alpha\right)$ cannot be given by considering only $y\left(\alpha\right)$ in our scheme due to the spatial information carried by the OFDM-IM subblocks. In the following, we investigate two different type of detection algorithms for the OFDM-IM scheme:

i) ML Detector: The ML detector for the OFDM-IM scheme considers all possible subblock realizations by searching for all possible subcarrier index combinations and signal constellation points in order to make a joint decision on the active indices and the constellation symbols for each subblock by minimizing the following metric:

$$\left(\hat{I}_\beta,\hat{\mathbf{s}}_\beta\right) = \arg\min_{I_\beta,\mathbf{s}_\beta}\sum_{\gamma=1}^{k}\left|y_F^\beta\left(i_{\beta,\gamma}\right) - h_F^\beta\left(i_{\beta,\gamma}\right)s_\beta\left(\gamma\right)\right|^2 \tag{15}$$

where $y_F^\beta\left(\xi\right)$ and $h_F^\beta\left(\xi\right)$ for $\xi = 1,\ldots,n$ are the received signals and the corresponding fading coefficients for the subblock β, i.e.,

$$\begin{aligned} y_F^\beta\left(\xi\right) &= y_F\left(n\left(\beta-1\right)+\xi\right) \\ h_F^\beta\left(\xi\right) &= h_F\left(n\left(\beta-1\right)+\xi\right). \end{aligned} \tag{16}$$

The total number of metric calculations performed in (15) is cM^k since I_β and $\mathbf{x}_\beta$ have c and M^k different realizations, respectively. Therefore, this ML detector becomes impractical for larger values of c and k due to its exponentially growing decoding complexity.

ii) Log-likelihood Ratio (LLR) Detector: The LLR detector of the OFDM-IM scheme provides the logarithm of the ratio of a posteriori probabilities of the frequency domain symbols by considering the fact that their values can be either non-zero or zero. This ratio, which is given below, gives information on the active status of the corresponding index for $\alpha = 1,\ldots,N$:

$$\lambda\left(\alpha\right) = \ln\frac{\sum_{\chi=1}^{M}P\left(x\left(\alpha\right) = s_\chi \mid y_F\left(\alpha\right)\right)}{P\left(x\left(\alpha\right) = 0 \mid y_F\left(\alpha\right)\right)} \tag{17}$$

where $s_\chi \in \mathcal{S}$. In other words, a larger $\lambda\left(\alpha\right)$ value means it is more probable that index α is selected by the index selector at the transmitter, i.e., it is active. Using Bayes' formula and considering that $\sum_{\chi=1}^{M}p\left(x\left(\alpha\right) = s_\chi\right) = k/n$ and $p\left(x\left(\alpha\right) = 0\right) = (n-k)/n$, (17) can be expressed as

$$\begin{aligned} \lambda\left(\alpha\right) = {} & \ln\left(k\right) - \ln\left(n-k\right) + \frac{\left|y_F\left(\alpha\right)\right|^2}{N_{0,F}} \\ & + \ln\left(\sum_{\chi=1}^{M}\exp\left(-\frac{1}{N_{0,F}}\left|y_F\left(\alpha\right) - h_F\left(\alpha\right)s_\chi\right|^2\right)\right). \end{aligned} \tag{18}$$

In order to prevent numerical overflow, the Jacobian logarithm [4] can be used in (18). As an example, for $k = n/2$ and binary-phase shift keying (BPSK) modulation, (18) simplifies to

$$\lambda\left(\alpha\right) = \max\left(a,b\right) + \ln\left(1 + \exp\left(-\left|b-a\right|\right)\right) + \frac{\left|y_F\left(\alpha\right)\right|^2}{N_{0,F}} \tag{19}$$

where

$$\begin{aligned} a &= -\left|y_F\left(\alpha\right) - h_F\left(\alpha\right)\right|^2/N_{0,F} \\ b &= -\left|y_F\left(\alpha\right) + h_F\left(\alpha\right)\right|^2/N_{0,F}. \end{aligned} \tag{20}$$

After calculation of the N LLR values, for each subblock, the receiver decides on k active indices out of them which have maximum LLR values. As seen from (18), the decoding complexity of this detector is linearly proportional to M

and comparable to that of classical OFDM. This detector is classified as near-ML since the receiver does not know the possible values of I_β. Although this is a desired feature for higher values of n and k, the detector can decide on a catastrophic set of active indices which is not included in I_β since $C(n,k) > c$ for $k > 1$, and $C(n,k) - c$ index combinations are unused at the transmitter.

After detection of the active indices by one of the detectors presented above at the receiver, the information is passed to the "index demapper", which performs the opposite action of the "index selector" block given in Fig. 1, to provide an estimate of the index-selecting p_1 bits. Demodulation of the constellation symbols is straightforward once the active indices are determined.

III. Implementation of the OFDM-IM Scheme

In this subsection, we focus on the index selector and index demapper blocks and provide different implementations of them. As stated in Section II, the index selector block maps the incoming bits to a combination of active indices out of $C(n,k)$ possible candidates, and the task of the index demapper is to provide an estimate of these bits by processing the detected active indices which are provided by either the ML or LLR OFDM-IM detector.

It is worth mentioning that the OFDM-IM scheme can be implemented without using a bit splitter at the beginning, i.e., by using a single group ($g = 1$) which results in $n = N$. However, in this case, $C(n,k)$ can take very large values which make the implementation of the overall system difficult. Therefore, instead of dealing with a single OFDM block with higher dimensions, we split this block into smaller subblocks to ease the index selection and detection processes at the transmitter and receiver sides, respectively. The following mappers are proposed for the new scheme:

i) Look-up Table Method: In this mapping method, a look-up table of size c is created to use at both transmitter and receiver sides. At the transmitter, this look-up table provides the corresponding indices for the incoming p_1 bits for each subblock, and it performs the opposite operation at the receiver. A look-up table example is presented in Table I for $n = 4, k = 2$, and $c = 4$, where $s_\chi, s_\zeta \in \mathcal{S}$. Since $C(4,2) = 6$, two combinations out of six are discarded. Although a very efficient and simple method for smaller c values, this mapping method is not feasible for higher values of n and k due to the size of the table. We employ this method with the ML detector since the receiver has to know the set of possible indices for ML decoding, i.e., it requires a look-up table. On the other hand, a look-up table cannot be used with an LLR detector since the receiver cannot decide on active indices if the detected indices do not exist in the table.

ii) Combinadics Method: The combinational number system (combinadics) provides a one-to-one mapping between natural numbers and k-combinations, for all n and k [5], [6], i.e., it maps a natural number to a strictly decreasing sequence

$$J = \{c_k, \ldots, c_1\} \tag{21}$$

TABLE I
A look-up table example for $n = 4, k = 2$ and $p_1 = 2$

Bits	*Indices*	*subblocks*
[0 0]	$\{1,2\}$	$\left[s_\chi \; s_\zeta \; 0 \; 0\right]^T$
[0 1]	$\{2,3\}$	$\left[0 \; s_\chi \; s_\zeta \; 0\right]^T$
[1 0]	$\{3,4\}$	$\left[0 \; 0 \; s_\chi \; s_\zeta\right]^T$
[1 1]	$\{1,4\}$	$\left[s_\chi \; 0 \; 0 \; s_\zeta\right]^T$

where $c_k > \cdots > c_1 \geq 0$. In other words, for fixed n and k, all $Z \in [0, C(n,k) - 1]$ can be presented by a sequence J of length k, which takes elements from the set $\{0, \ldots, n-1\}$ according to the following equation:

$$Z = C(c_k, k) + \cdots + C(c_2, 2) + C(c_1, 1). \tag{22}$$

As an example, for $n = 8, k = 4$, $C(8,4) = 70$, the following J sequences can be calculated:

$$\begin{aligned}
69 &= C(7,4)+C(6,3)+C(5,2)+C(4,1) \rightarrow J=\{7,6,5,4\}\\
68 &= C(7,4)+C(6,3)+C(5,2)+C(3,1) \rightarrow J=\{7,6,5,3\}\\
&\vdots\\
32 &= C(6,4)+C(5,3)+C(4,2)+C(1,1) \rightarrow J=\{6,5,4,1\}\\
31 &= C(6,4)+C(5,3)+C(4,2)+C(0,1) \rightarrow J=\{6,5,4,0\}\\
&\vdots\\
1 &= C(4,4)+C(2,3)+C(1,2)+C(0,1) \rightarrow J=\{4,2,1,0\}\\
0 &= C(3,4)+C(2,3)+C(1,2)+C(0,1) \rightarrow J=\{3,2,1,0\}.
\end{aligned}$$

The algorithm, which finds the lexicographic ordered J sequences for all n, can be explained as follows: start by choosing the maximal c_k that satisfies $C(c_k, k) \leq Z$, and then choose the maximal c_{k-1} that satisfies $C(c_{k-1}, k-1) \leq Z - C(c_k, k)$ and so on [6]. In our scheme, for each subblock, we first convert the p_1 bits entering the index selector to a decimal number Z, and then feed this decimal number to the combinadics algorithm to select the active indices as $J + 1$. At the receiver side, after determining active indices, we can easily get back to the decimal number $\hat{Z}$ using (22). We then apply this number to a p_1-bit decimal-to-binary converter. We employ this method with the LLR detector for higher c values to avoid look-up tables. However, it can give a catastrophic result at the exit of the decimal-to-binary converter if $\hat{Z} \geq c$; nevertheless, we use this detector for the increased bit-rate.

IV. Performance Analysis of the OFDM-IM Scheme

In this section, we analytically evaluate the average bit error probability (ABEP) of the OFDM-IM scheme using the ML decoder with a look-up table.

The channel coefficients in the frequency domain are related to the coefficients in the time domain by

$$\mathbf{h}_F = \mathbf{W}_N \mathbf{h}_T^0 \tag{23}$$

where $\mathbf{h}_T^0$ is the zero-padded version of the vector $\mathbf{h}_T$ with length N, i.e.,

$$\mathbf{h}_T^0 = \left[h_T(1) \; \ldots \; h_T(\nu) \; 0 \; \ldots \; 0\right]^T. \tag{24}$$

It can easily be shown that $h_F(\alpha), \alpha = 1, \ldots, N$ follows the distribution $\mathcal{CN}(0,1)$, since taking the Fourier transform of a Gaussian vector gives another Gaussian vector. However, the elements of $\mathbf{h}_F$ are no longer uncorrelated. The correlation matrix of $\mathbf{h}_F$ is given as

$$\mathbf{K} = E\left\{\mathbf{h}_F\mathbf{h}_F^H\right\} = \mathbf{W}_N E\left\{\mathbf{h}_T^0 {\mathbf{h}_T^0}^H\right\}\mathbf{W}_N^H = \mathbf{W}_N^H \tilde{\mathbf{I}}\mathbf{W}_N \tag{25}$$

where

$$\tilde{\mathbf{I}} = \begin{bmatrix} \frac{1}{\nu}\mathbf{I}_{\nu\times\nu} & \mathbf{0}_{\nu\times(N-\nu)} \\ \mathbf{0}_{(N-\nu)\times\nu} & \mathbf{0}_{(N-\nu)\times(N-\nu)} \end{bmatrix}_{N\times N} \tag{26}$$

is an all-zero matrix except for its first ν diagonal elements which are all equal to $\frac{1}{\nu}$. It should be noted that $\mathbf{K}$ becomes a diagonal matrix if $\nu = N$, which is very unlikely for a practical OFDM scheme. Nevertheless, since $\mathbf{K}$ is a Hermitian Toeplitz matrix, the pairwise error probability (PEP) events within different subblocks are identical, and it is sufficient to investigate the PEP events within a single subblock to determine the overall system performance. Without loss of generality, we can choose the first subblock, and introduce the following matrix notation for the input-output relationship in the frequency domain:

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{w} \tag{27}$$

where $\mathbf{y} = \left[y_F(1)\cdots y_F(n)\right]^T$, $\mathbf{X}$ is an $n\times n$ all-zero matrix except for its main diagonal elements which are $x(1), \ldots, x(n)$, $\mathbf{h} = \left[h_F(1)\cdots h_F(n)\right]^T$ and $\mathbf{w} = \left[w_F(1)\cdots w_F(n)\right]^T$. Let us define

$$\mathbf{K}_n = E\left\{\mathbf{h}\mathbf{h}^H\right\}. \tag{28}$$

In fact, this is an $n\times n$ submatrix centered along the main diagonal of the matrix $\mathbf{K}$. Thus it is valid for all subblocks. If $\mathbf{X}$ is transmitted and it is erroneously detected as $\hat{\mathbf{X}}$, as we know that the receiver can make decision errors on both active indices and constellation symbols, the well-known conditional pairwise error probability (CPEP) expression for the model in (27) is given as [7]

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\middle|\mathbf{h}\right) = Q\left(\sqrt{\delta/(2N_{0,F})}\right) \tag{29}$$

where $\delta = \left\|(\mathbf{X}-\hat{\mathbf{X}})\mathbf{h}\right\|_F^2 = \mathbf{h}^H\mathbf{A}\mathbf{h}$ and

$$\mathbf{A} = \left(\mathbf{X}-\hat{\mathbf{X}}\right)^H\left(\mathbf{X}-\hat{\mathbf{X}}\right). \tag{30}$$

We can approximate $Q(x)$ quite well using [8]

$$Q(x) \cong \frac{1}{12}e^{-x^2/2} + \frac{1}{4}e^{-2x^2/3}. \tag{31}$$

Thus, the unconditional PEP (UPEP) of the OFDM-IM scheme can be obtained by

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\right) \cong E_{\mathbf{h}}\left\{\frac{1}{12}\exp(-q_1\delta) + \frac{1}{4}\exp(-q_2\delta)\right\} \tag{32}$$

where $q_1 = 1/(4N_{0,F})$ and $q_2 = 1/(3N_{0,F})$. Let

$$r_1 = \text{rank}(\mathbf{K}_n). \tag{33}$$

Since $r_1 < n$ for our scheme, we use the spectral theorem [9] to calculate the expectation above on defining $\mathbf{K}_n = \mathbf{Q}\mathbf{D}\mathbf{Q}^H$ and $\mathbf{h} = \mathbf{Q}\mathbf{u}$, where $E\left\{\mathbf{u}\mathbf{u}^H\right\} = \mathbf{D}$ is an $r_1\times r_1$ diagonal matrix. Considering

$$\delta = \mathbf{u}^H\mathbf{Q}^H\mathbf{A}\mathbf{Q}\mathbf{u} \tag{34}$$

and the p.d.f. of $\mathbf{u}$ given by

$$f(\mathbf{u}) = \frac{\pi^{-r_1}}{\det(\mathbf{D})}\exp\left(-\mathbf{u}^H\mathbf{D}^{-1}\mathbf{u}\right) \tag{35}$$

the UPEP can be calculated as

$$\begin{aligned} &P\left(\mathbf{X}\to\hat{\mathbf{X}}\right) \\ &\cong \frac{\pi^{-r_1}}{12\det(\mathbf{D})}\int_{\mathbf{u}}\exp\left(-\mathbf{u}^H\left[\mathbf{D}^{-1}+q_1\mathbf{Q}^H\mathbf{A}\mathbf{Q}\right]\mathbf{u}\right)d\mathbf{u} \\ &+ \frac{\pi^{-r_1}}{4\det(\mathbf{D})}\int_{\mathbf{u}}\exp\left(-\mathbf{u}^H\left[\mathbf{D}^{-1}+q_2\mathbf{Q}^H\mathbf{A}\mathbf{Q}\right]\mathbf{u}\right)d\mathbf{u} \quad (36) \\ &= \frac{1/12}{\det(\mathbf{I}_{r_1}+q_1\mathbf{D}\mathbf{Q}^H\mathbf{A}\mathbf{Q})} + \frac{1/4}{\det(\mathbf{I}_{r_1}+q_2\mathbf{D}\mathbf{Q}^H\mathbf{A}\mathbf{Q})} \quad (37) \\ &= \frac{1/12}{\det(\mathbf{I}_n+q_1\mathbf{Q}\mathbf{D}\mathbf{Q}^H\mathbf{A})} + \frac{1/4}{\det(\mathbf{I}_n+q_2\mathbf{Q}\mathbf{D}\mathbf{Q}^H\mathbf{A})} \quad (38) \\ &= \frac{1/12}{\det(\mathbf{I}_n+q_1\mathbf{K}_n\mathbf{A})} + \frac{1/4}{\det(\mathbf{I}_n+q_2\mathbf{K}_n\mathbf{A})} \quad (39) \end{aligned}$$

where (36) and (37) are related via (35), and (38) is obtained from the identity

$$\det(\mathbf{I}_{r_1}+\mathbf{M}\mathbf{N}) = \det(\mathbf{I}_n+\mathbf{N}\mathbf{M}) \tag{40}$$

where the dimensions of $\mathbf{M}$ and $\mathbf{N}$ are $r_1\times n$ and $n\times r_1$, respectively. We have the following remarks:

Remark 1: Let us define

$$\mathbf{A}_i = \mathbf{I}_n + q_i\mathbf{K}_n\mathbf{A} = \mathbf{I}_n + q_i\mathbf{B} \tag{41}$$

for $i = 1, 2$. Since

$$\det(\mathbf{A}_i) = \prod\nolimits_{\xi=1}^{n}\lambda_\xi(\mathbf{A}_i) = \prod\nolimits_{\xi=1}^{r}\left(1+q_i\lambda_\xi(\mathbf{B})\right) \tag{42}$$

where $r = \text{rank}(\mathbf{B})$, for high SNR values $(q_i \gg 1)$, we can rewrite (39) as

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\right) \backsimeq \left(12q_1^r\prod_{\xi=1}^{r}\lambda_\xi(\mathbf{B})\right)^{-1} + \left(4q_2^r\prod_{\xi=1}^{r}\lambda_\xi(\mathbf{B})\right)^{-1}. \tag{43}$$

As seen from this result, the diversity order of the system is determined by r, which is upper bounded according to the rank inequality [9] by

$$r \le \min\{r_1, r_2\} \tag{44}$$

where $r_2 = \text{rank}(\mathbf{A})$. On the other hand we have $\min r_2 = 1$ when the receiver correctly detects all of the active indices and makes a single decision error out of k M-ary symbols. It can be shown that r can take values from the interval $[1, n]$.

Remark 2: In order to improve the diversity order of the system, we can by-pass the M-ary modulations by setting $p = p_1$ and only transmit data with the indices of the active subcarriers at the expense of reduced bit rate, since we always

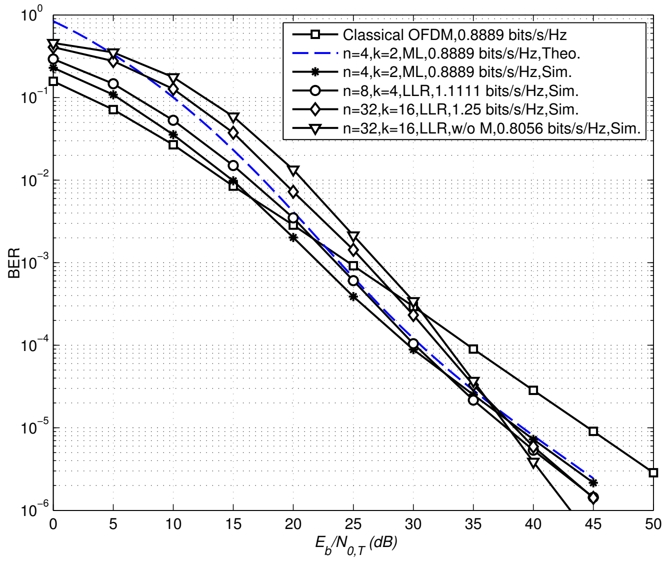

Fig. 2. BER performance of OFDM-IM with different configurations

guarantee $r_2 \geq 2$ without M-ary symbol errors.

After the evaluation of the UPEP from (39), the ABEP of the OFDM-IM can be evaluated by

$$P_b \approx \frac{1}{p\, n_{\mathbf{X}}} \sum_{\mathbf{X}} \sum_{\hat{\mathbf{X}}} P\left(\mathbf{X} \rightarrow \hat{\mathbf{X}}\right) e\left(\mathbf{X}, \hat{\mathbf{X}}\right) \qquad (45)$$

where $n_{\mathbf{X}}$ is the number of the possible realizations of $\mathbf{X}$ and $e(\mathbf{X}, \hat{\mathbf{X}})$ represents the number of bit errors for the corresponding pairwise error event.

V. Simulation Results

In this section, we present simulation results for the OFDM-IM scheme with different configurations and make comparisons with classical OFDM. BER performance of these schemes was evaluated with Monte Carlo simulations. In all simulations, we assumed the following system parameters: $N = 128$, $\nu = 10$ and $L = 16$. Furthermore, we employed a BPSK constellation ($M = 2$) for all systems.

As seen from Fig. 2, at a BER value of 10^{-5} our new scheme with $n = 4, k = 2$ achieves approximately 6 dB better BER performance than classical OFDM operating at the same spectral efficiency. This significant improvement in BER performance can be explained by the improved distance spectrum of the OFDM-IM scheme, where higher diversity orders are obtained for the bits carried by the active indices. For comparison, the theoretical curve obtained from (45) is also depicted in the same figure for the $n = 4, k = 2$ scheme, which uses an ML decoder. As seen from Fig. 2, the theoretical curve becomes very tight with the computer simulation curve with increasing SNR values. For higher values of n, we employ the combinadics method for the index mapping and demapping operations with the LLR decoder. We observe that despite their increased data rates, $n = 8, k = 4$ and $n = 32, k = 16$ OFDM-IM schemes exhibits close BER performance to the low-rate $n = 4, k = 2$ OFDM scheme. This can be explained by the fact that for high SNR, the error performance of the OFDM-IM scheme is dominated by the PEP events with $r = 1$ as we discussed in the previous section.

In Fig. 2, we also show the BER performance of the OFDM-IM scheme which does not employ M-ary modulations ($n = 32, k = 16$, LLR, w/o M), and relies on the transmission of data with subcarrier indices only. As seen from Fig. 2, this scheme achieves a diversity order of two, as proved in Section IV, and exhibits the best BER performance for high SNR values with a slight decrease in the spectral efficiency compared to classical OFDM employing BPSK modulation.

VI. Conclusion

A novel OFDM scheme, which uses the indices of the active subcarriers to transmit data, has been proposed in this paper. It has been shown that the proposed scheme achieves significantly better BER performance than classical OFDM. As future research, we believe that the implementation of different transceiver structures could be realized for the OFDM-IM scheme to increase the data rate as well as to improve the error performance. The proposed scheme should also be investigated in real world conditions such as mobility.

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