Performance of Spatial Modulation in the Presence of Channel Estimation Errors

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***Abstract*—This work investigates the negative effects of channel estimation errors on the performance of spatial modulation (SM) when operating over flat Rayleigh fading channels. The pairwise error probability of the SM scheme is derived in the presence of channel estimation errors and an upper bound on the average bit error probability is evaluated for M-PSK and M-QAM signalling. It is shown via computer simulations that the derived upper bound becomes very tight with increasing signal-to-noise ratio (SNR) and the SM scheme is quite robust to channel estimation errors.**

***Index Terms*—Channel estimation errors, MIMO systems, spatial modulation.**

I. Introduction

SPATIAL modulation (SM), which exploits the indices of multiple transmit antennas as an additional source of information besides the conventional M-ary signal constellations, is a promising multiple-input multiple-output (MIMO) transmission technique that has been recently proposed [1]. It has been shown in [2] and [3] that SM can achieve better error performance than V-BLAST (Vertical-Bell Lab Layered Space-Time) in some cases under the assumption that perfect channel state information (P-CSI) is available at the receiver. However, in practical applications, we hardly have P-CSI at the receiver, and a channel estimator is employed to provide unknown channel parameters. Therefore, it is important to assess the system performance in the presence of imperfect CSI before choosing the appropriate channel estimation technique.

The effects of channel estimation errors on the performance of SM and space-shift keying (SSK) modulation [4], a special version of SM in which only antenna indices are exploited to convey information, have been investigated by some researchers [4–8]. In fact, the authors of [5] emphasized that the conventional SM/SSK modulations are based on P-CSI, and a degradation in performance is unavoidable when these systems are subject to imperfect CSI. In [4], [5] and [6] the performance of SM and SSK were studied in the presence of imperfect CSI only by computer simulations. In [7], the authors have studied the performance of SSK with partial CSI using analytical methods. More recently, the same authors have extended their analyses in [8] by examining the error performance of SSK with imperfect channel knowledge in detail. However, to the best of our knowledge, the error performance of SM with imperfect CSI has not been investigated through analytical methods before, and in this work, we aim to shed light on this timely and interesting topic.

In this letter, we provide an analytical approach for the calculation of the average bit error probability (ABEP) of SM with imperfect CSI. First, the pairwise error probability (PEP) of SM is derived for general M-ary constellations; then, an asymptotically tight upper bound on the ABEP is provided. Our computer simulations indicate that the derived upper bounds become very tight with increasing signal-to-noise ratio (SNR) and SM is quite robust to imperfect CSI compared to V-BLAST. The rest of the letter is organized as follows. In Section II, the considered system model is given. Our analytical approach to the ABEP calculation of SM is presented in Section III. Numerical examples are provided in Section IV. Finally, conclusions are given in Section V.

Notation: Bold capital letters are used for matrices. $\Re\{x\}$ denotes the real part of the complex variable x. The probability of an event is denoted by $P(\cdot)$. For a random variable (r.v.) X, $E\{X\}$, $Var\{X\}$ and $M_X(t)$ denote the mean, variance and moment generating function (MGF) of X, respectively. $X \sim \mathcal{CN}\left(0,\sigma_X^2\right)$ represents the distribution of a circularly symmetric complex Gaussian r.v with variance σ_X^2. $Q(\cdot)$ denotes the tail probability of the standard Gaussian distribution.

II. System Model

We consider a MIMO system operating over a quasi-static Rayleigh flat fading channel with n_T transmit and n_R receive antennas. The channel fading coefficient between the tth transmit and the rth receive antenna, denoted by $\alpha_{t,r}$, is distributed as $\mathcal{CN}(0,1)$.

Assume that $\log_2(Mn_T)$ information bits enter the SM transmitter at each transmission interval. The transmitter specifies the identity of the active transmit antenna by using the first $\log_2(n_T)$ bits of the incoming bit stream, then maps the remaining $\log_2(M)$ bits onto the corresponding M-ary signal constellation. Therefore, according to the SM technique, during each transmission interval, only one transmit antenna, which transmits an M-ary constellation symbol s, is active. As an example, for $M=4$, $n_T=4$, four information bits are transmitted by the transmitter at each signalling interval, where the first two bits determine the index of the active transmit antenna, while the last two bits determine the quadrature phase shift keying (QPSK) symbol that is transmitted through this active antenna.

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The spatially modulated symbol is denoted by $x = (i, s)$, where s is transmitted over the ith transmit antenna. The received signal at the rth receive antenna $(r = 1, \cdots, n_R)$ is given by

$$y_r = \alpha_{i,r} s + w_r \tag{1}$$

where w_r is a sample of additive white Gaussian noise with distribution $\mathcal{CN}(0, N_0)$. Assuming the SM symbol $x = (i, s)$ is transmitted and it is erroneously detected as $\hat{x} = (j, \hat{s})$, when CSI is perfectly known at the receiver, the conditional pairwise error probability (CPEP) is given by [9]

$$P(x \to \hat{x} \mid \mathbf{H}) = Q\left(\sqrt{\frac{\gamma}{2}\sum_{r=1}^{n_R}\left|\alpha_{i,r}s - \alpha_{j,r}\hat{s}\right|^2}\right) \tag{2}$$

where $\mathbf{H} = [\alpha_{t,r}]_{n_T \times n_R}$ is the channel matrix with independent and identically distributed entries and $\gamma = E\{|s|^2\}/N_0$ is the average SNR at each receiver antenna.

In practical systems, a channel estimator at the receiver provides the fading coefficient estimates $\beta_{t,r}$. If the channel is estimated with least squares (LS), the estimation error model has the form $\beta_{t,r} = \alpha_{t,r} + \epsilon_{t,r}$, where $\epsilon_{t,r}$ represents the channel estimation error which is independent of $\alpha_{t,r}$, and is distributed according to $\mathcal{CN}\left(0, \sigma_\epsilon^2\right)$ [10]. Consequently, the distribution of $\beta_{t,r}$ becomes $\mathcal{CN}\left(0, 1+\sigma_\epsilon^2\right)$, and $\beta_{t,r}$ is dependent on $\alpha_{t,r}$ with the correlation coefficient $\rho = 1/\sqrt{1+\sigma_\epsilon^2}$, i.e, when $\sigma_\epsilon^2 \to 0$, then $\rho \to 1$. We assume that ρ is known at the receiver. In this work, two different scenarios are considered: i) fixed σ_ϵ^2: the value of the estimation error is fixed for all SNR values in order to determine the pure effect of the imperfect channel knowledge on the error performance, and ii) variable σ_ϵ^2: the value of the estimation error is adjusted in accordance with the SNR as $\sigma_\epsilon^2 = 1/(\gamma N)$, where N depends on the number of pilot symbols used in training and the chosen estimation method [11].

In the presence of channel estimation errors, assuming the SM symbol $x = (i, s)$ is transmitted, the mean and variance of the received signal $y_r, r = 1, \cdots, n_R$ conditioned on $\beta_{i,r}$ are given as [12]

$$\begin{aligned} E\{y_r \mid \beta_{i,r}\} &= \rho^2 \beta_{i,r} s \\ Var\{y_r \mid \beta_{i,r}\} &= N_0 + \left(1-\rho^2\right)|s|^2. \end{aligned} \tag{3}$$

Thus, the optimal receiver of the SM decides in favor of the symbol $\hat{s}$ and transmit antenna index j that minimizes the following metric for an M-ary signal constellation

$$\begin{aligned} (j, \hat{s}) = \arg\min_{i,s} \sum_{r=1}^{n_R} \Bigg(& \frac{\left|y_r - \rho^2\beta_{i,r}s\right|^2}{N_0 + (1-\rho^2)|s|^2} \\ & + \ln\left(N_0 + \left(1-\rho^2\right)|s|^2\right)\Bigg) \end{aligned} \tag{4}$$

to maximize the a posteriori probability of $y_r, r = 1, \cdots, n_R$, which are complex Gaussian r.v.'s. Note that for constellations with constant envelope $\left(|s|^2 = 1, \forall s\right)$ such as M-ary PSK (M-PSK), the metric in (4) reduces to

$$(j, \hat{s}) = \arg\min_{i,s} \sum_{r=1}^{n_R}\left|y_r - \rho^2\beta_{i,r}s\right|^2. \tag{5}$$

III. Pairwise Error Probability Calculation

In this section, first, we evaluate the PEP of SM for M-PSK with imperfect CSI, then we generalize the analysis to M-ary quadrature amplitude modulation (M-QAM). After the evaluation of PEP, ABEP expressions will be provided for the SM scheme.

A. ABEP of the SM for M-PSK

Assuming $x = (i, s)$ is transmitted, the probability of deciding in favor of $\hat{x} = (j, \hat{s})$ is given from (5) as

$$\begin{aligned} P(x \to \hat{x} \mid \hat{\mathbf{H}}) = P\Big(&\sum_{r=1}^{n_R}\left|y_r - \rho^2\beta_{j,r}\hat{s}\right|^2 \\ &< \sum_{r=1}^{n_R}\left|y_r - \rho^2\beta_{i,r}s\right|^2\Big) \end{aligned} \tag{6}$$

where $\hat{\mathbf{H}} = [\beta_{t,r}]_{n_T \times n_R}$ is the estimated channel matrix. After simple manipulation, we obtain

$$\begin{aligned} P(x \to \hat{x} \mid \hat{\mathbf{H}}) = P\Big(&\sum_{r=1}^{n_R} \rho^4|\beta_{i,r}|^2 - \rho^4|\beta_{j,r}|^2 \\ &- 2\rho^2\Re\{y_r^*(\beta_{i,r}s - \beta_{j,r}\hat{s})\} > 0\Big) = P(D > 0) \end{aligned} \tag{7}$$

where the sum is denoted by D. Considering (3), we observe that D is a Gaussian r.v. with

$$\begin{aligned} E\{D\} &= -\rho^4\sum_{r=1}^{n_R}|\beta_{i,r}s - \beta_{j,r}\hat{s}|^2 \\ Var\{D\} &= 2\rho^4\left(N_0 + \left(1-\rho^2\right)\right)\sum_{r=1}^{n_R}|\beta_{i,r}s - \beta_{j,r}\hat{s}|^2 \end{aligned}$$

Thus, the conditional PEP (CPEP) of SM can be written as

$$P(x \to \hat{x} \mid \hat{\mathbf{H}}) = Q\left(\rho^2\sqrt{\frac{\sum_{r=1}^{n_R}|\beta_{i,r}s - \beta_{j,r}\hat{s}|^2}{2\left(N_0 + (1-\rho^2)\right)}}\right). \tag{8}$$

Using an alternative form of the Gaussian Q-function [13], (8) can be rewritten as

$$\begin{aligned} &P(x \to \hat{x} \mid \hat{\mathbf{H}}) = \\ &\frac{1}{\pi}\int_0^{\pi/2}\exp\left(\frac{-\rho^4\sum_{r=1}^{n_R}|\beta_{i,r}s - \beta_{j,r}\hat{s}|^2}{4\sin^2\theta\left(N_0 + (1-\rho^2)\right)}\right)d\theta. \end{aligned} \tag{9}$$

Defining $d_r \triangleq |\beta_{i,r}s - \beta_{j,r}\hat{s}|^2$, we derive its MGF from [14] as

$$M_{d_r}(t) = \frac{1}{1 - \lambda\left(1+\sigma_\epsilon^2\right)t} \tag{10}$$

where

$$\lambda = \begin{cases} 2, & \text{if } i \neq j \\ |s - \hat{s}|^2, & \text{if } i = j. \end{cases} \tag{11}$$

Finally, integrating (9) over the probability density function (p.d.f.) of d_r and using (10), the unconditional PEP (UPEP) of the SM scheme is obtained as follows:

$$P(x \to \hat{x}) = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\lambda\rho^2}{4(N_0 + (1-\rho^2))}}\right)^{n_R} d\theta \tag{12}$$

which has a closed form solution provided in [13]. We observe from (12) that, when compared to the SM scheme with P-CSI, the same diversity order of n_R is asymptotically attained for values of ρ approaching unity.

After the evaluation of the UPEP, the ABEP of the SM scheme can be upper bounded by the following asymptotically tight union bound [13]:

$$P_b \leq \frac{1}{2^k} \sum_{n=1}^{2^k} \sum_{m=1}^{2^k} \frac{P\left(x_n \rightarrow x_m\right) e_{n,m}}{k} \tag{13}$$

where $\{x_n\}_{n=1}^{2^k}$ is the set of all possible SM symbols, $k = \log_2\left(Mn_T\right)$ is the number of information bits per SM symbol, and $e_{n,m}$ is the number of bit errors associated with the corresponding PEP event.

It is worth mentioning that the PEP expression provided in (12) can be generalized to SSK modulation, which does not use amplitude/phase modulations, by taking $\lambda = 2$ in (12).

B. ABEP of the SM for M-QAM

In order to determine the UPEP of the SM using M-QAM signalling in the presence of channel estimation errors, we consider the mismatched maximum likelihood (ML) receiver that uses the ML decision metric of the P-CSI case by replacing $\alpha_{t,r}$ by $\beta_{t,r}$. This is mainly due to the fact the decision metric given in (4) for constellations with non-constant envelope is quite complicated to analyse.

The decision metric for the mismatched ML receiver is given as

$$(j, \hat{s}) = \arg\min_{i,s} \sum\nolimits_{r=1}^{n_R} \left|y_r - \beta_{i,r} s\right|^2. \tag{14}$$

Then the CPEP of the SM scheme is obtained by

$$\begin{aligned} P(x \rightarrow \hat{x} \mid \hat{\mathbf{H}}) = P\Big(& \sum\nolimits_{r=1}^{n_R} \left|\beta_{i,r}\right|^2 |s|^2 - \left|\beta_{j,r}\right|^2 |\hat{s}|^2 \\ & - 2\Re\left\{y_r^* \left(\beta_{i,r} s - \beta_{j,r} \hat{s}\right)\right\} > 0\Big) = P\left(D > 0\right) \end{aligned} \tag{15}$$

where the sum is denoted by D, which is a Gaussian r.v. with

$$\begin{aligned} E\{D\} = \sum\nolimits_{r=1}^{n_R} & \left|\beta_{i,r}\right|^2 |s|^2 \left(1 - 2\rho^2\right) - \left|\beta_{j,r}\right|^2 |\hat{s}|^2 \\ & + 2\rho^2 2\Re\left\{\beta_{i,r} s - \beta_{j,r} \hat{s}\right\} \\ Var\{D\} = 2(N_0 + & \left(1 - \rho^2\right) |s|^2) \sum\nolimits_{r=1}^{n_R} \left|\beta_{i,r} s - \beta_{j,r} \hat{s}\right|^2. \end{aligned}$$

On defining $\tilde{D} = \rho^2 D$, and taking $\left(1 + \sigma_\epsilon^2\right)^2 \approx \left(1 + \sigma_\epsilon^2\right)$ for $\sigma_\epsilon^2 \ll 1$, which is quite reasonable for practical applications, we have

$$\begin{aligned} E\{\tilde{D}\} &\approx -\rho^2 \sum\nolimits_{r=1}^{n_R} \left|\beta_{i,r} s - \beta_{j,r} \hat{s}\right|^2 \\ Var\{\tilde{D}\} &= 2\rho^4 (N_0 + \left(1 - \rho^2\right) |s|^2) \sum\nolimits_{r=1}^{n_R} \left|\beta_{i,r} s - \beta_{j,r} \hat{s}\right|^2 \end{aligned}$$

which yields the approximate CPEP expression

$$P(x \rightarrow \hat{x} \mid \hat{\mathbf{H}}) \approx Q\left(\sqrt{\frac{\sum_{r=1}^{n_R} \left|\beta_{i,r} s - \beta_{j,r} \hat{s}\right|^2}{2\left(N_0 + \left(1 - \rho^2\right) |s|^2\right)}}\right). \tag{16}$$

The MGF of $d_r \triangleq \left|\beta_{i,r} s - \beta_{j,r} \hat{s}\right|^2$ is again given by (10) while $\lambda = |s|^2 + |\hat{s}|^2$ if $i \neq j$ and $\lambda = |s - \hat{s}|^2$ if $i = j$, for this case. Finally, the UPEP of SM is calculated for M-QAM as

$$P(x \rightarrow \hat{x}) \approx \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\lambda}{4\left(N_0 + \left(1 - \rho^2\right) |s|^2\right)}}\right)^{n_R} d\theta. \tag{17}$$

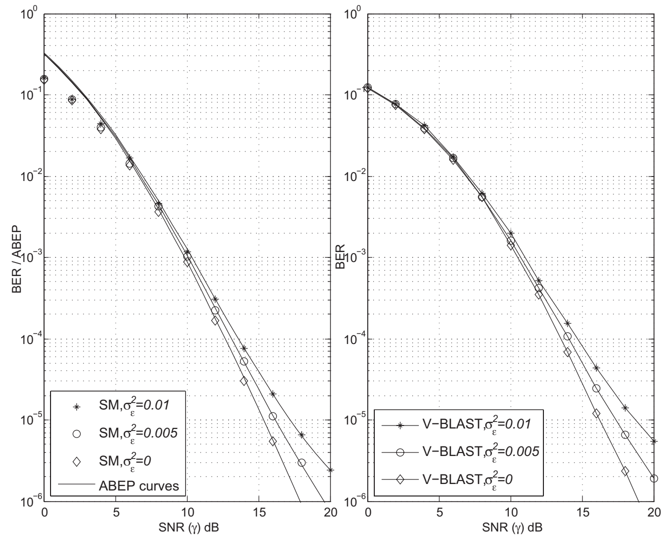

Fig. 1. BER performance of SM with $n_T = 4$, QPSK and V-BLAST with $n_T = 4$, BPSK (4 bits/s/Hz) with optimal receivers (fixed σ_ϵ^2).

Then, the union bound given in (13) can be still used to evaluate the approximate ABEP of the SM scheme for M-QAM.

IV. Simulation Results

In this section, the bit error rate (BER) performance of the SM and V-BLAST schemes with imperfect CSI is evaluated via Monte Carlo simulations with respect to the average SNR per receive antenna, and the results are compared with the analytical results of (13) for QPSK and 16-QAM. In all simulations, it was assumed that $n_R = 4$. The natural mapping was applied for both antenna and signal constellation points. According to Section II, the power of the estimation error $\left(\sigma_\epsilon^2\right)$ was either fixed (to 0.01, 0.007, 0.005 and 0.003 values) for all SNR values in order to determine the pure effect of the estimation error on the performance, or was adjusted according to the SNR values by taking N as 1, 3 and 10. For comparison purposes, the performance of the P-CSI case $\left(\sigma_\epsilon^2 = 0\right)$ is also included.

In Fig. 1, computer simulation results are presented for the SM scheme with $n_T = 4$ and QPSK, and V-BLAST with $n_T = 4$ and binary PSK (BPSK) at 4 bits/s/Hz for fixed σ_ϵ^2 values. Both schemes use optimal ML receivers. As a reference, the corresponding ABEP upper bound curves are also shown with solid lines for the SM scheme. First, as seen from Fig. 1, the theoretical upper bounds provided by (13) become extremely tight with increasing SNR for all σ_ϵ^2 values. Second, we observe that the SM scheme is resistant to channel estimation errors for values of $\sigma_\epsilon^2 \leq 0.01$, and furthermore it is more robust than V-BLAST at this spectral efficiency. As an example, at a BER value of 10^{-5}, the SNR degradation of SM is 0.9 dB for $\sigma_\epsilon^2 = 0.005$ $(\rho = 0.9975)$ compared to the P-CSI case, while the degradation for V-BLAST is observed as 1.1 dB, which is slightly higher than SM. In Fig. 2, computer simulation results are presented for the same systems given in Fig. 1 at 4 bits/s/Hz for variable σ_ϵ^2 values. As seen from this figure, for this configuration, SM and V-BLAST are closely

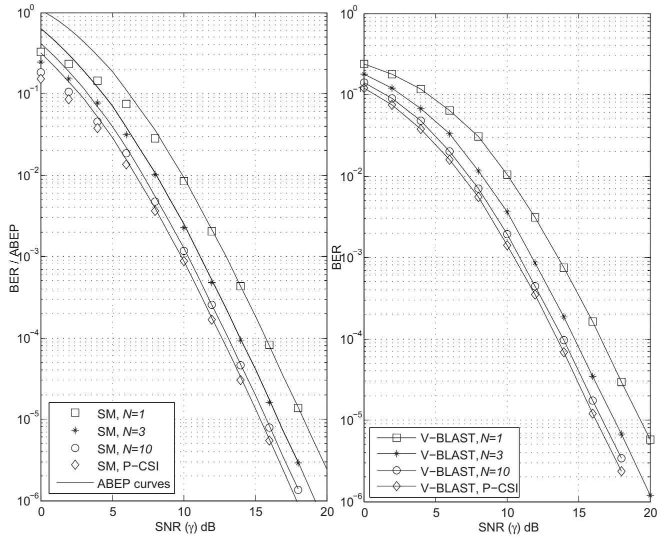

Fig. 2. BER performance of SM with $n_T = 4$, QPSK and V-BLAST with $n_T = 4$, BPSK (4 bits/s/Hz) with optimal receivers (variable σ_ϵ^2).

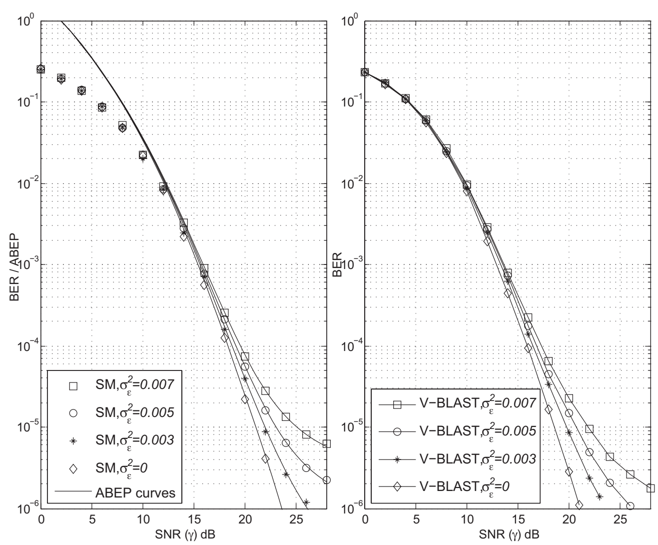

Fig. 3. BER performance of SM with $n_T = 4$, 16-QAM and V-BLAST with $n_T = 3$, QPSK (6 bits/s/Hz) with mismatched receivers (fixed σ_ϵ^2).

matched again, since at a BER value of 10^{-5} the degradation amounts for these systems are observed as 0.4 dB, 1.3 dB and 3 dB for SM and, 0.5 dB, 1.3 dB and 3.1 dB for V-BLAST, compared to the P-CSI case for $N = 10$, 3 and 1 values, respectively.

Simulation results are depicted in Fig. 2 for the SM scheme with $n_T = 4$ and 16-QAM, and V-BLAST with $n_T = 3$ and QPSK at 6 bits/s/Hz, with the corresponding ABEP upper bound curves for SM for fixed σ_ϵ^2 values. For this case, both schemes use mismatched ML receivers. As seen from Fig. 2, although the approximation given in (17) is made in this case, the ABEP upper bound curves are still very tight with increasing SNR values. At a BER value of 10^{-5}, when compared with the P-CSI case, the degradation amount in SNR is observed for the SM case as 0.9 dB and 2.1 dB for $\sigma_\epsilon^2 = 0.003$ and 0.005, respectively, while these values are equal to 1.2 dB and 2.3 dB for V-BLAST. Therefore, we conclude that SM is more robust to channel estimation errors than V-BLAST for reasonable channel estimation error values.

It is worth mentioning that by considering Figs. 1-3, we observe that the SM scheme is quite robust to channel estimation errors compared to V-BLAST, which has a higher ML decoding complexity and a higher implementation cost due to requirement of inter-antenna synchronization and multiple radio frequency (RF) chains at the transmitter.

V. Conclusions

In this letter, we have investigated the error performance of SM with imperfect CSI. The UPEP of SM has been derived for general M-ary signal constellations, and an upper bound on ABEP has been provided which is shown to become very tight with increasing SNR. It has been observed that the SM scheme is quite robust to channel estimation errors compared to V-BLAST. Therefore, we conclude that the SM scheme could be considered as a competitive alternative to V-BLAST in practical applications.

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