

# Reliable Two-Path Successive Relaying

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**Abstract**—Emerging two-path successive relaying protocols generally rely on error-free source-relay channels and/or interference-free inter-relay channels to achieve high-rate and full-diversity. In this paper, by removing these optimistic assumptions, a novel two-path successive relaying scheme that benefits from relay selection and distributed space-time block coding (STBC), and transfers the data from the source to the destination via relays in a reliable fashion is proposed. The proposed scheme can achieve full diversity without the requirement of perfect decoding at relays since not only the destination but also the relays benefit from distributed STBC and relay selection. As the target STBC, coordinate interleaved orthogonal design (CIOD) for two transmit antennas is considered. The average symbol error probability of the proposed scheme is derived and its error performance is compared with reference systems.

## I. INTRODUCTION

Two-path successive relaying has emerged recently as a promising cooperative transmission strategy since it provides significant bandwidth efficiency improvements over the classical relaying methods [1]. Two-path successive relaying has been realized by exploiting distributed space-time block codes (STBC) [2], which create a virtual multi antenna scheme for STBC transmission. An effective distributed STBC for two-path relaying has been proposed in [3] which provides full-rate and full-diversity; however, this code does not permit single symbol detection, which makes its implementation difficult and costly. More recently, a new distributed STBC, which uses the coordinate interleaved orthogonal design (CIOD) [4], has been proposed for two-path relaying [5]. This scheme provides single symbol decoding and achieves better error performance than the scheme of [3] and provides full-diversity when *i*) the relays perfectly decode and forward the signals transmitted from the source, and *ii*) the channel between relays is not affected by fading; however, these are extremely optimistic assumptions which cannot hold for practical wireless networks.

In this paper, by removing these two assumptions, we propose a novel two-path successive relaying scheme which can reliably transfer the data from the source to the destination via relays under realistic network conditions. The main contributions of this work are as follows: 1) It is shown that full-diversity can be achieved with the proposed protocol at both relay nodes and the destination without any assumptions. This is accomplished by implementing distributed CIOD signaling at the destination and at one of the relays, while the other relay benefits from relay selection; 2) The average symbol error probability (ASEP) of the proposed scheme is evaluated analytically for *M*-ary quadrature amplitude modulation (*M*-QAM). Taking into account the error propagation, it is shown

that analytical and computer simulation results match very well; 3) It is shown that for realistic network conditions, the proposed scheme achieves significantly better bit error rate (BER) performance than its counterparts given in the literature.\*

## II. SYSTEM MODEL AND THE NEW SCHEME

We consider a relay network with a single source node *S*, two relay nodes *R*<sub>1</sub> and *R*<sub>2</sub> and a destination node *D*. Each node has a single antenna and operates in half-duplex mode. We assume that there is no direct transmission from *S* to *D*.  $h_{SR_i}$  and  $h_{R_iD}$ ,  $i = 1, 2$ , represent the wireless channel fading coefficients between *S* and relays, and relays and *D*, respectively, while the inter-relay channel  $h_R$  is reciprocal. We assume that the real and imaginary parts of  $h_{SR_i}$ ,  $h_{R_iD}$  and  $h_R$  follow the  $\mathcal{N}(0, 1/2)$  distribution, and  $h_{R_iD}$ ,  $i = 1, 2$ , are known at *D*, while  $h_{SR_i}$  and  $h_R$  are known at the relays. The variance of the zero-mean complex Gaussian noise samples  $n_{R_1}(t)$ ,  $n_{R_2}(t)$  and  $n_D(t)$  at relays and *D* is assumed to be  $N_0$ , where  $t$  ( $t = 1, 2, 3$ ) represents time slots.

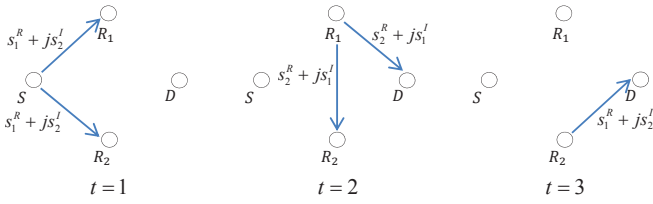
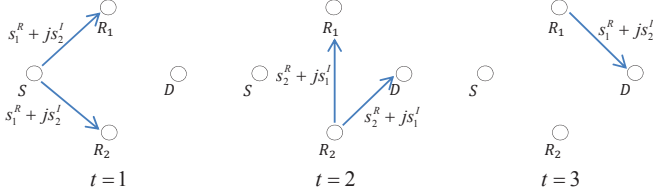
The proposed scheme is based on CIOD transmission with two transmit antennas which may be presented by either of the following  $2 \times 2$  transmission matrices [4]:

$$\begin{bmatrix} s_1^R + js_2^I & 0 \\ 0 & s_2^R + js_1^I \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} s_2^R + js_1^I & 0 \\ 0 & s_1^R + js_2^I \end{bmatrix} \quad (2)$$

where columns and rows correspond to time slots and transmit antennas, respectively, and  $s_1$  and  $s_2$  are two complex information symbols drawn from a rotated *M*-QAM constellation. Assuming that a square *M*-QAM constellation with signal points  $s = s^R + js^I$  where  $s^R, s^I \in \{\pm 1, \pm 3, \dots, \pm\sqrt{M}-1\}$  is rotated by an angle  $\theta$ , the rotated signal constellation symbols are denoted by  $s_\theta = se^{j\theta} = s_\theta^R + js_\theta^I$  whose real and imaginary components  $s_\theta^R$  and  $s_\theta^I$  take *M* distinct values from the set  $\{s^R \cos \theta - s^I \sin \theta\}$ , where  $\theta = 31.7^\circ$  is the optimal rotation angle for square *M*-QAM [4]. Consequently, for a given  $s_\theta^R$  ( $s_\theta^I$ ),  $s_\theta^I$  ( $s_\theta^R$ ) can be determined uniquely. As an example, for 4-QAM, the rotated symbols with distinct real and imaginary parts are  $s_\theta \in \{(-1.376 + j0.325), (-0.325 - j1.376), (0.325 + j1.376), (1.376 - j0.325)\}$ , and if  $s_\theta^R = -1.376$  is given for this constellation, we know that  $s_\theta^I =$

\*Notation: For a complex variable  $s = s^R + js^I$ ,  $s^R$  and  $s^I$  denote the real and imaginary parts of  $s$ , where  $j = \sqrt{-1}$ .  $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$  denotes the Gaussian distribution of a real r.v.  $X$  with mean  $\mu_X$  and variance  $\sigma_X^2$ .


 Fig. 1. Three Phase Successive Relaying with Stronger  $S-R_1$  Channel

 Fig. 2. Three Phase Successive Relaying with Stronger  $S-R_2$  Channel

0.325. Constellation rotation is required for CIODs to achieve full-diversity, while in our scheme it is also required to identify symbols from their real or imaginary parts.

In the proposed protocol given in Figs. 1-2, within every consecutive three time slots, two information symbols  $s_1$  and  $s_2$  drawn from a rotated  $M$ -QAM constellation are transmitted from  $S$  as follows: In the first time slot,  $S$  processes  $s_1$  and  $s_2$ , and transmits the coordinate interleaved symbol  $s_1^R + js_2^I$  to  $R_1$  and  $R_2$ . If the  $S-R_1$  channel is stronger than the  $S-R_2$  channel,  $R_1$  decodes  $s_1^R$  and  $s_2^I$  first, i.e., it obtains  $s_1$  and  $s_2$  since each symbol can be identified from its real or imaginary part only, which take distinct values after the constellation rotation. Then  $R_1$  forms and transmits the coordinate interleaved symbol  $s_2^R + js_1^I$  to  $R_2$  and  $D$  in the second time slot. As seen from (1), distributed CIOD signaling is achieved for  $R_2$  after two time slots. In the third time slot, after detecting  $s_1$  and  $s_2$ ,  $R_2$  transmits  $s_1^R + js_2^I$  to  $D$  to create the virtual multiple-input single-output (MISO) system using the CIOD matrix given in (2) for  $D$ . As seen from Fig. 2, similar procedures can be applied when the  $S-R_2$  channel is stronger than the  $S-R_1$  channel. Note that a virtual MISO system is created for both  $D$  and one of the relays, while the other relay benefits from relay selection. Therefore, the overall diversity order of the system becomes two since not only  $D$ , but also  $R_1$  and  $R_2$  achieve a diversity order of two. On the other hand, the transmission rate of the proposed scheme is  $2/3$  symbols per channel use since only two information symbols are transmitted in three time slots.

### III. SYMBOL ERROR PROBABILITY (SEP) ANALYSIS OF THE PROPOSED SCHEME

In this section, we evaluate the ASEP of the proposed scheme for general  $M$ -QAM. Without loss of generality, we can analyze the error performance of the signaling scheme given in Fig. 1 where the  $S-R_1$  channel is stronger than the  $S-R_2$  channel since the ASEP is the same for both cases. The average destination SEP of the scheme given in Fig. 1 can be expressed as  $P_D = \frac{1}{M} \sum_s \sum_{\hat{s}} P_D(s \rightarrow \hat{s})$  for  $s \neq \hat{s}$  where  $P_D(s \rightarrow \hat{s})$  stands for the pairwise error probability (PEP) at the destination associated with detection of symbol  $\hat{s}$  given that symbol  $s$  is transmitted.

Destination PEP  $P_D(s \rightarrow \hat{s})$  can be expressed as the sum of four probabilities related to the error events at the relays as  $P_D(s \rightarrow \hat{s}) = P_1 + P_2 + P_3 + P_4$  where

$$P_1 = P_{R_1}^c(s) P_{R_2}^c(s | R_1^c) P_D(s \rightarrow \hat{s} | R_1^c, R_2^c)$$

$$P_2 = \sum_{\bar{s}, \bar{s} \neq s} P_{R_1}^c(s) P_{R_2}^c(s \rightarrow \bar{s} | R_1^c) P_D(s \rightarrow \hat{s} | R_1^c, R_2^e)$$

$$P_3 = \sum_{\bar{s}, \bar{s} \neq s} P_{R_1}^e(s \rightarrow \bar{s}) P_{R_2}^c(s | R_1^e) P_D(s \rightarrow \hat{s} | R_1^e, R_2^c)$$

$$P_4 = \sum_{\bar{s}, \bar{s} \neq s} \sum_{\tilde{s}, \tilde{s} \neq \bar{s}} P_{R_1}^e(s \rightarrow \tilde{s}) P_{R_2}^e(s \rightarrow \bar{s} | R_1^e) P_D(s \rightarrow \hat{s} | R_1^e, R_2^e)$$

in which  $P_{R_1}^c(s)$  is the probability of correct detection of  $s$  at  $R_1$ ,  $P_{R_2}^c(s | R_1^c)$  is the correct detection probability of  $s$  at  $R_2$  conditioned on correct detection at  $R_1$ ,  $P_D(s \rightarrow \hat{s} | R_1^c, R_2^c)$  is the PEP at the destination conditioned on the correct detection of  $s$  at both relays,  $P_{R_2}^e(s \rightarrow \bar{s} | R_1^c)$  is the PEP at  $R_2$  conditioned on the correct detection of  $s$  at  $R_1$ ,  $P_D(s \rightarrow \hat{s} | R_1^e, R_2^e)$  is the PEP at the destination conditioned on correct detection of  $s$  at  $R_1$  and erroneous detection of  $s$  to  $\bar{s}$  at  $R_2$ ,  $P_{R_1}^e(s \rightarrow \tilde{s})$  is the PEP at  $R_1$ ,  $P_{R_2}^c(s | R_1^e)$  is the probability of correct detection of  $s$  at  $R_2$  conditioned on the erroneous detection of  $s$  to  $\tilde{s}$  at  $R_1$ ,  $P_D(s \rightarrow \hat{s} | R_1^e, R_2^c)$  is the PEP at the destination conditioned on correct detection of  $s$  at  $R_2$  and erroneous detection of  $s$  to  $\tilde{s}$  at  $R_1$ ,  $P_{R_2}^e(s \rightarrow \bar{s} | R_1^e)$  is the PEP at  $R_2$  conditioned on the erroneous detection of  $s$  to  $\tilde{s}$  at  $R_1$ ,  $P_D(s \rightarrow \hat{s} | R_1^e, R_2^e)$  is the PEP at the destination conditioned on the erroneous detection of  $s$  at both relays. Our analyses show that the ASEP at the destination is dominated by the case in which  $\tilde{s} = \bar{s} = \hat{s}$ , i.e., for the case where successive identical erroneous detections occur in the relaying scheme. Therefore we obtain the following:

$$P_2 \cong P_{R_1}^c(s) P_{R_2}^e(s \rightarrow \hat{s} | R_1^c) P_D(s \rightarrow \hat{s} | R_1^c, R_2^e)$$

$$P_3 \cong P_{R_1}^e(s \rightarrow \hat{s}) P_{R_2}^c(s | R_1^e) P_D(s \rightarrow \hat{s} | R_1^e, R_2^c)$$

$$P_4 \cong P_{R_1}^e(s \rightarrow \hat{s}) P_{R_2}^e(s \rightarrow \hat{s} | R_1^e) P_D(s \rightarrow \hat{s} | R_1^e, R_2^e)$$

Our analyses also show that the ASEP at the destination is mainly dominated by  $P_1$ ,  $P_2$  and  $P_4$ , and the effect of  $P_3$  can be ignored. This can be explained by the fact that  $P_{R_2}^c(s | R_1^e) \ll P_{R_2}^e(s \rightarrow \hat{s} | R_1^e)$  while  $P_D(s \rightarrow \hat{s} | R_1^e, R_2^c) \sim P_D(s \rightarrow \hat{s} | R_1^e, R_2^e)$ . As we will show in the sequel, independent of the SNR, the probability of symbol error increases dramatically at  $R_2$  when  $R_1$  makes a decision error. Therefore, the probability  $P_{R_2}^c(s | R_1^e)$  can be neglected when compared to  $P_{R_2}^e(s \rightarrow \hat{s} | R_1^e)$ , and we can assume  $P_3 \ll P_4$ . Therefore the ASEP can be rewritten as the sum of the three terms as

$$P_D(s \rightarrow \hat{s}) \cong P_{R_1}^c(s) P_{R_2}^c(s | R_1^c) P_D(s \rightarrow \hat{s} | R_1^c, R_2^c) + P_{R_1}^c(s) P_{R_2}^e(s \rightarrow \hat{s} | R_1^c) P_D(s \rightarrow \hat{s} | R_1^c, R_2^e) + P_{R_1}^e(s \rightarrow \hat{s}) P_{R_2}^e(s \rightarrow \hat{s} | R_1^e) P_D(s \rightarrow \hat{s} | R_1^e, R_2^e). \quad (3)$$

In order to obtain the ASEP, we calculated (3) for  $s = s_1$ . The derivation of each term in (3) is quite lengthy. In Appendices A, B and C, the details of the derivations are given for the terms related with  $R_1$  ( $P_{R_1}^c(s)$ ,  $P_{R_1}^e(s \rightarrow \hat{s})$ ),  $R_2$  ( $P_{R_2}^c(s | R_1^c)$ ,  $P_{R_2}^e(s \rightarrow \hat{s} | R_1^c)$ ,  $P_{R_2}^e(s \rightarrow \hat{s} | R_1^e)$ ) and  $D$  ( $P_D(s \rightarrow \hat{s} | R_1^c, R_2^c)$ ,  $P_D(s \rightarrow \hat{s} | R_1^c, R_2^e)$ ,  $P_D(s \rightarrow \hat{s} | R_1^e, R_2^e)$ ), respectively.

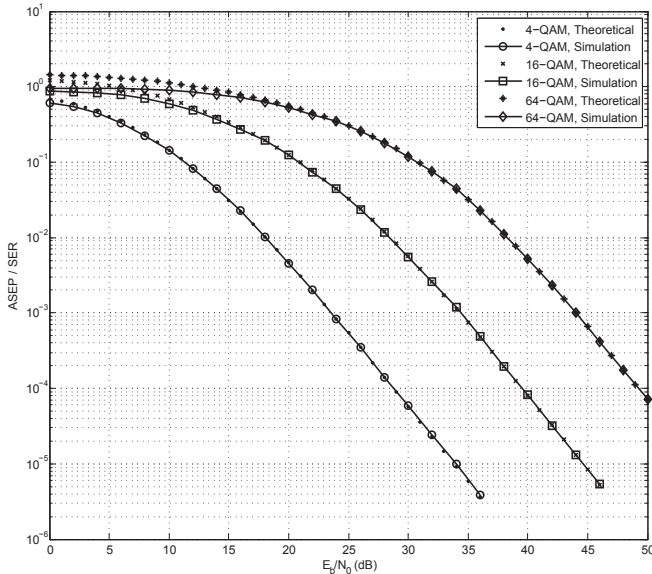


Fig. 3. Theoretical performance curves for the new scheme

#### IV. NUMERICAL RESULTS

In this section, we present numerical results based on the above analytical expressions as well as Monte Carlo simulation results for quasi-static uncorrelated Rayleigh fading channels.

In Fig. 3, we show the theoretical ASEP curves and computer simulation results for the proposed scheme using 4-QAM, 16-QAM and 64-QAM modulation schemes. As seen from Fig. 3, with increasing SNR, the theoretical curves become extremely tight with the computer simulation curves, and therefore support our analytical results.

In Fig. 4, we compare the bit error rate performance of the proposed scheme with the scheme of [5] and the classical CIOD (i.e., the scheme of [5] with error-free relays) which provides a performance benchmark. We employ 4-QAM modulation for all systems. For the new scheme and the scheme of [5], we consider realistic network conditions in which relays can make erroneous detections. As seen from Fig. 4, without perfect decoding at relays, the scheme of [5] cannot achieve full diversity while the proposed scheme does. We also observe from Fig. 4 that the reference distributed CIOD with error-free relays and the proposed scheme achieve the same diversity order; however, the difference in the error performance can be explained by the additional errors at the relays which increase the overall ASEP at the destination of the proposed scheme.

#### V. CONCLUSION

A novel reliable two-path successive relaying protocol has been proposed and its error performance has been investigated comprehensively. It has been shown that by the achievement of full-diversity at all of the nodes of the network, it is possible to transfer data from the source to the destination via relays in a reliable manner.

#### APPENDIX A

##### CALCULATION OF PROBABILITIES RELATED TO $R_1$

$$P_{R_1}^c(s), P_{R_1}^e(s \rightarrow \hat{s})$$

As we mentioned earlier, we consider the case in which the  $S-R_1$  channel is stronger than the  $S-R_2$  channel, i.e.,  $h_1 >$

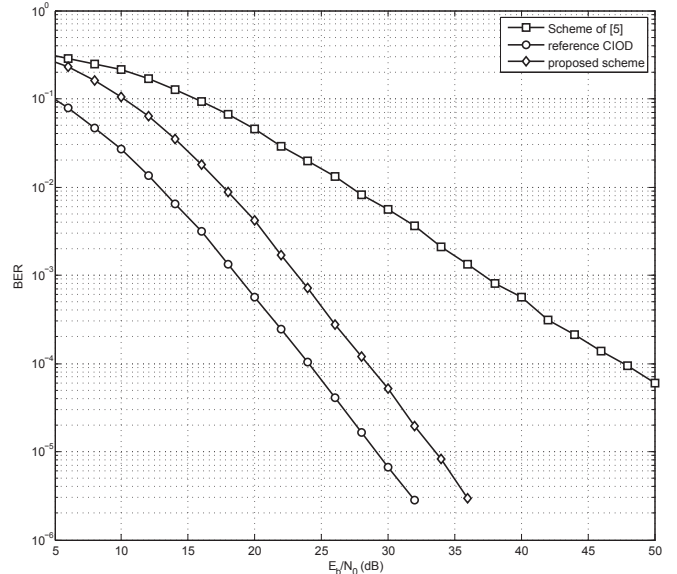


Fig. 4. Performance of the new scheme, the CIOD and the scheme of [5]

$h_2$ , where  $h_1 = |h_{SR_1}|^2$  and  $h_2 = |h_{SR_2}|^2$ . On analyzing the order statistics we obtain  $f_{h_1}(h_1) = 2(1 - e^{-h_1})e^{-h_1}$ ,  $h_1 > 0$  and  $f_{h_2}(h_2) = 2e^{-2h_2}$ ,  $h_2 > 0$  [6].

$i) P_{R_1}^c(s_1)$ : The received signal at  $R_1$  for  $t = 1$  is given as  $r_{R_1}(1) = h_{SR_1}(s_1^R + js_2^I) + n_{R_1}(1)$ . Thanks to coordinate interleaving, for the independent detection of  $s_1^R$ ,  $R_1$  obtains

$$y_{R_1} = h_{SR_1}^R r_{R_1}^R(1) + h_{SR_1}^I r_{R_1}^I(1) = h_1 s_1^R + w_{R_1} \quad (4)$$

where  $w_{R_1} \triangleq h_{SR_1}^R n_{R_1}^R(1) + h_{SR_1}^I n_{R_1}^I(1)$ . Therefore, the detection problem of  $s_1^R$  becomes the detection of a modified  $M$ -PAM signal subject to fading. Let us denote the possible values of  $s_1^R$  in ascending order as  $a_1, a_2, \dots, a_M$ . Considering the transmission model given in (4), we have  $M$  decision intervals separated by the threshold values (normalized by  $h_1$ )  $\lambda_1, \lambda_2, \dots, \lambda_{M-1}$ . As an example for 4-QAM, we have  $\lambda_1 = -\cos\theta$ ,  $\lambda_2 = 0$  and  $\lambda_3 = \cos\theta$ . Considering that  $w_{R_1}$  in (4) is distributed as  $\mathcal{N}(0, \psi^2)$ , where  $\psi^2 = \sigma^2 h_1$  and  $\sigma^2 = N_0/2$ , the correct detection probability for  $a_i$ ,  $i = 1, \dots, M$ , conditioned on  $h_1$  can be written as

$$P_{R_1}^c(a_i) \Big|_{h_1} = \int_a^b f_{y_{R_1}}(y_{R_1} | a_i, h_1) dy_{R_1}$$

where  $a = -\infty$ ,  $b = h_1 \lambda_1$  and  $a = h_1 \lambda_{i-1}$ ,  $b = h_1 \lambda_i$  for  $i = 1, M$  and  $2 \leq i \leq M-1$ , respectively, and conditioned on  $a_i$  and  $h_1$ ,  $y_{R_1}$  follows the  $\mathcal{N}(h_1 a_i, \psi^2)$  distribution for all  $i$ . Simple manipulation gives

$$P_{R_1}^c(a_i) \Big|_{h_1} = \begin{cases} 1 - Q\left(\frac{h_1(\lambda_1 - a_i)}{\psi}\right), & i = 1, i = M \\ 1 - Q\left(\frac{h_1(a_i - \lambda_{i-1})}{\psi}\right) - Q\left(\frac{h_1(\lambda_i - a_i)}{\psi}\right), & \text{o.w.} \end{cases}$$

Using the alternative form of the  $Q$ -function

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2 \sin^2 \theta}\right) d\theta \quad (5)$$

and considering the moment generating function (m.g.f.) of  $h_1$  given as  $M_{h_1}(s) = 2/(2 - 3s + s^2)$ , the unconditional

correct detection probability for  $a_i$  is obtained as follows:

$$P_{R_1}^c(a_i) = \begin{cases} 1 - q\left(\frac{(\lambda_1 - a_1)^2}{N_0}\right), & i = 1, i = M \\ 1 - q\left(\frac{(a_i - \lambda_{i-1})^2}{N_0}\right) - q\left(\frac{(\lambda_i - a_i)^2}{N_0}\right), & 2 \leq i \leq M-1 \end{cases}$$

where

$$q(x) \triangleq \frac{1}{\pi} \int_0^{\pi/2} M_{h_1}\left(\frac{-x}{\sin^2 \theta}\right) d\theta = \frac{1}{2} \left(1 - \frac{2}{\sqrt{1 + \frac{1}{x}}} + \frac{1}{\sqrt{1 + \frac{2}{x}}}\right). \quad (6)$$

ii)  $P_{R_1}^e(s_1 \rightarrow \hat{s}_1)$ : For the signaling scheme of (4), by the integration of the conditional p.d.f. of  $y_{R_1}$  over the decision intervals mentioned above, the exact conditional PEP (CPEP) can be written in general form as

$$P_{R_1}^e(a_i \rightarrow a_j) \Big|_{h_1} = \int_a^b f_{y_{R_1}}(y_{R_1} | a_i, h_1) dy_{R_1} \quad (7)$$

where  $(a, b)$  pairs are given by  $(-\infty, h_1 \lambda_1)$ ,  $(h_1 \lambda_{j-1}, h_1 \lambda_j)$  and  $(h_1 \lambda_{M-1}, \infty)$  for  $j = 1, 2 \leq j \leq M-1$  and  $j = M$ , respectively for  $1 \leq i \leq M$ . Using (5),  $M_{h_1}(s)$  and (6), the unconditional PEP (UPEP) is derived as follows (for  $i = 1, \dots, M, j \neq i$ ):

$$P_{R_1}^e(a_i \rightarrow a_j) = \begin{cases} q\left(\frac{(a_i - \lambda_1)^2}{N_0}\right), & j = 1 \\ \left| q\left(\frac{(\lambda_{j-1} - a_i)^2}{N_0}\right) - q\left(\frac{(\lambda_j - a_i)^2}{N_0}\right) \right|, & \text{o.w.} \\ q\left(\frac{(\lambda_{M-1} - a_i)^2}{N_0}\right), & j = M. \end{cases}$$

Since the coordinate interleaving technique allows us to distinguish symbols from only their real (or imaginary) parts,  $P_{R_1}^c(a_i) = P_{R_1}^c(s_1)$  and  $P_{R_1}^e(a_i \rightarrow a_j) = P_{R_1}^e(s_1 \rightarrow \hat{s}_1)$ , for  $s_1 \neq \hat{s}_1$ .

## APPENDIX B

### CALCULATION OF PROBABILITIES RELATED TO $R_2$

$$P_{R_2}^c(s | R_1^c), P_{R_2}^e(s \rightarrow \hat{s} | R_1^c), P_{R_2}^e(s \rightarrow \hat{s} | R_1^e)$$

i)  $P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^e)$ : Assuming that  $s_1 = s_1^R + j s_1^I$  has been erroneously detected at  $R_1$  as  $\tilde{s}_1 = \tilde{s}_1^R + j \tilde{s}_1^I$ , the received signals at  $R_2$  at the first two time slots can be written as

$$\begin{bmatrix} r_{R_2}(1) \\ r_{R_2}(2) \end{bmatrix} = \begin{bmatrix} s_1^R + j s_2^I & 0 \\ 0 & s_2^R + j \tilde{s}_1^I \end{bmatrix} \begin{bmatrix} h_{SR_2} \\ h_R \end{bmatrix} + \begin{bmatrix} n_{R_2}(1) \\ n_{R_2}(2) \end{bmatrix}.$$

$R_2$  applies CIOD detection procedures to decode  $s_1$  and  $s_2$  by combining the received signals as  $\gamma_{R_2} = \alpha_{R_2}^R + j \beta_{R_2}^I$  and  $\delta_{R_2} = \beta_{R_2}^R + j \alpha_{R_2}^I$  where  $\alpha_{R_2} = h_{SR_2}^* r_{R_2}(1)$  and  $\beta_{R_2} = h_{SR_2}^* r_{R_2}(2)$ . Then, the maximum likelihood (ML) decision rules for  $s_1$  and  $s_2$  are given as follows [4]:  $\hat{s}_i = \arg \min_{s_i} \{m_{R_2}(s_i)\}$ ,  $i = 1, 2$ , where

$$m_{R_2}(s_1) = h_3 (\gamma_{R_2}^R - h_2 s_1^R)^2 + h_2 (\gamma_{R_2}^I - h_3 s_1^I)^2$$

$$m_{R_2}(s_2) = h_2 (\delta_{R_2}^R - h_3 s_2^R)^2 + h_3 (\delta_{R_2}^I - h_2 s_2^I)^2$$

with  $h_2 \triangleq |h_{SR_2}|^2$  and  $h_3 \triangleq |h_R|^2$ . The CPEP of detecting  $\hat{s}_1$  when  $s_1$  is transmitted can be given as

$$P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^e) \Big|_{h_2, h_3} = P(m_{R_2}(\hat{s}_1) < m_{R_2}(s_1)). \quad (8)$$

Considering  $\gamma_{R_2} = h_2 s_1^R + j h_3 \tilde{s}_1^I + w_{R_2}^R(1) + j w_{R_2}^I(2)$ , where  $w_{R_2}(1) = h_{SR_2}^* n_{R_2}(1)$  and  $w_{R_2}(2) = h_R^* n_{R_2}(2)$ , we obtain

$$P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^e) \Big|_{h_2, h_3} = P\left(h_3 [h_2 \Delta_1 + w_{R_2}^R(1)]^2 + h_2 [h_3 \Delta_2 + w_{R_2}^I(2)]^2 < h_3 (w_{R_2}^R(1))^2 + h_2 [h_3 \Delta_3 + w_{R_2}^I(2)]^2\right) \quad (9)$$

where  $\Delta_1 \triangleq s_1^R - \tilde{s}_1^R$ ,  $\Delta_2 \triangleq \tilde{s}_1^I - s_1^I$  and  $\Delta_3 \triangleq \tilde{s}_1^I - s_1^I$ . After manipulation, we arrive at

$$P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^e) \Big|_{h_2, h_3} = P(A < 0) \quad (10)$$

where  $A \sim \mathcal{N}(\mu_A, \sigma_A^2)$ ,  $\mu_A = h_2 h_3 (h_2 \Delta_1^2 + h_3 (\Delta_2^2 - \Delta_3^2))$  and  $\sigma_A^2 = 4\sigma^2 h_2^2 h_3^2 (h_2 \Delta_1^2 + h_3 (\Delta_2 - \Delta_3)^2)$ . From (10), the desired CPEP expression can be obtained as

$$P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^e) \Big|_{h_2, h_3} = Q\left(\frac{h_2 \Delta_1^2 + h_3 (\Delta_2^2 - \Delta_3^2)}{2\sigma \sqrt{h_2 \Delta_1^2 + h_3 (\Delta_2 - \Delta_3)^2}}\right). \quad (11)$$

Let us define  $\Phi_1 = s_1^R - \tilde{s}_1^R$  and  $\Phi_2 = s_1^I - \tilde{s}_1^I$ . As we mentioned earlier, the average SEP at the destination is dominated by the case where  $\tilde{s}_1 = \hat{s}_1$ . For this case we have  $\Delta_1^2 = \Phi_1^2$ ,  $\Delta_2 = 0$  and  $\Delta_3 = \Phi_2^2$  in (11), which gives

$$P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^e) \Big|_{h_2, h_3} = Q\left(\frac{h_2 \Phi_1^2 - h_3 \Phi_2^2}{2\sigma \sqrt{h_2 \Phi_1^2 + h_3 \Phi_2^2}}\right). \quad (12)$$

Please note that (12) can take very large values when  $h_3 \Phi_2^2 > h_2 \Phi_1^2$ , which supports our assumption that  $P_{R_2}^e(s | R_1^e) \ll P_{R_2}^e(s \rightarrow \hat{s} | R_1^e)$ . Thus, we can approximate the UPEP for high SNR after tedious calculations as

$$P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^e) \approx P(h_2 \Phi_1^2 < h_3 \Phi_2^2) = \frac{2\Phi_2^2}{\Phi_1^2 + 2\Phi_2^2} \quad (13)$$

which proves that the probability of error becomes very high at  $R_2$  if  $R_1$  forwards an erroneously detected signal.

ii)  $P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^c)$ : With correct detection at  $R_1$ , the CPEP of detection  $\hat{s}_1$  when  $s_1$  is transmitted can be obtained by setting  $\tilde{s}_1^I = s_1^I$  in (11) ( $\Delta_2^2 = \Phi_2^2, \Delta_3 = 0$ ) as

$$P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^c) \Big|_{h_2, h_3} = Q\left(\sqrt{\frac{h_2 \Phi_1^2 + h_3 \Phi_2^2}{2N_0}}\right) \quad (14)$$

which is the CPEP of the classical CIOD. Let us define  $u = u_1 + u_2$  where  $u_1 \triangleq h_2 \Phi_1^2$  and  $u_2 \triangleq h_3 \Phi_2^2$ . Using (5) and defining  $M_u(s) = E\{e^{su}\}$ , the corresponding UPEP can be evaluated as follows:

$$P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^c) = \frac{1}{\pi} \int_0^{\pi/2} M_u\left(\frac{-1}{4N_0 \sin^2 \theta}\right) d\theta. \quad (15)$$

Considering  $f_{u_1}(u_1) = (2/\Phi_1^2) e^{-2u_1/\Phi_1^2}$  and  $f_{u_2}(u_2) = (1/\Phi_2^2) e^{-u_2/\Phi_2^2}$  the p.d.f. of  $u$ , which is the sum of two exponential r.v.'s, can be calculated by [6]

$$f_u(u) = \int_0^u f_{u_2}(u_2) f_{u_1}(u - u_2) du_2 = \frac{2(e^{-2u/\Phi_1^2} - e^{-u/\Phi_2^2})}{(\Phi_1^2 - 2\Phi_2^2)}$$

Then, the corresponding m.g.f. is evaluated as

$$M_u(s) = \frac{2}{(\Phi_1^2 s - 2)(\Phi_2^2 s - 1)}. \quad (16)$$

Combining (15) and (16), we obtain

$$P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^c) = \frac{\Phi_1^2(1 - \rho_1) + 2\Phi_2^2(-1 + \rho_2)}{2(\Phi_1^2 - 2\Phi_2^2)} \quad (17)$$

where  $\rho_1 \triangleq 1/\sqrt{1 + 8N_0/\Phi_1^2}$  and  $\rho_2 \triangleq 1/\sqrt{1 + 4N_0/\Phi_2^2}$ .

**iii)**  $P_{R_2}^c(s_1 | R_1^c)$ : The correct detection probability of  $s_1$  at  $R_2$  given that  $R_1$  forwarded the correct  $s_1$  component can be easily obtained by using (17) in the following union bound:

$$P_{R_2}^c(s_1 | R_1^c) \leq 1 - \frac{1}{M} \sum_{\hat{s}_1, \hat{s}_1 \neq s_1} P_{R_2}^e(s_1 \rightarrow \hat{s}_1 | R_1^c). \quad (18)$$

#### APPENDIX C

##### CALCULATION OF PROBABILITIES RELATED TO $D$

$$P_D(s \rightarrow \hat{s} | R_1^c, R_2^c), P_D(s \rightarrow \hat{s} | R_1^c, R_2^e), P_D(s \rightarrow \hat{s} | R_1^e, R_2^e)$$

**i)**  $P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^e)$ : Assuming that  $s_1 = s_1^R + js_1^I$  has been erroneously detected as  $\hat{s}_1 = \hat{s}_1^R + j\hat{s}_1^I$  and  $\bar{s}_1 = \bar{s}_1^R + j\bar{s}_1^I$  at  $R_1$  and  $R_2$ , respectively, the received signals at the destination for  $t = 2$  and  $3$  can be expressed as

$$\begin{bmatrix} r_D(3) \\ r_D(2) \end{bmatrix} = \begin{bmatrix} \bar{s}_1^R + js_1^I & 0 \\ 0 & s_2^R + js_2^I \end{bmatrix} \begin{bmatrix} h_{R_2D} \\ h_{R_1D} \end{bmatrix} + \begin{bmatrix} n_D(3) \\ n_D(2) \end{bmatrix}. \quad (19)$$

According to the CIOD detection procedures, after processing and interleaving these signals as  $\alpha_D = h_{R_2D}^* r_D(3)$ ,  $\beta_D = h_{R_1D}^* r_D(2)$  and  $\gamma_D = \alpha_D^R + j\beta_D^I$ ,  $\delta_D = \beta_D^R + j\alpha_D^I$ , the receiver calculates the ML decision metrics as

$$\begin{aligned} \hat{s}_1 &= \arg \min_{s_1} \left\{ h_4 (\gamma_D^R - h_5 s_1^R)^2 + h_5 (\gamma_D^I - h_4 s_1^I)^2 \right\} \\ \hat{s}_2 &= \arg \min_{s_2} \left\{ h_5 (\delta_D^R - h_4 s_2^R)^2 + h_4 (\delta_D^I - h_5 s_2^I)^2 \right\} \end{aligned}$$

where  $h_4 \triangleq |h_{R_1D}|^2$  and  $h_5 \triangleq |h_{R_2D}|^2$ . Considering  $\gamma_D = h_5 \bar{s}_1^R + jh_4 \bar{s}_1^I + w_D^R(3) + jw_D^I(2)$  where  $w_D(3) = h_{R_2D}^* n_D(3)$  and  $w_D(2) = h_{R_1D}^* n_D(2)$ , similar to (8), the CPEP can be calculated as

$$\begin{aligned} P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^e) \Big|_{h_4, h_5} &= \\ P \left( h_4 [h_5 \Delta_4 + w_D^R(3)]^2 + h_5 [h_4 \Delta_2 + w_D^I(2)]^2 \right. \\ &< \left. h_4 [h_5 \Delta_5 + w_D^R(3)]^2 + h_5 [h_4 \Delta_3 + w_D^I(2)]^2 \right) \quad (20) \end{aligned}$$

where  $\Delta_2$  and  $\Delta_3$  are defined in (9), and  $\Delta_4 \triangleq \bar{s}_1^R - \hat{s}_1^R$ ,  $\Delta_5 \triangleq \bar{s}_1^R - s_1^R$ . After simple calculations, we obtain

$$\begin{aligned} P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^e) \Big|_{h_4, h_5} &= \\ &= Q \left( \frac{h_5 (\Delta_4^2 - \Delta_5^2) + h_4 (\Delta_2^2 - \Delta_3^2)}{2\sigma \sqrt{h_5 (\Delta_4 - \Delta_5)^2 + h_4 (\Delta_2 - \Delta_3)^2}} \right). \quad (21) \end{aligned}$$

As mentioned earlier, we assume that for the dominant case  $\bar{s}_1^R = \hat{s}_1^R$  and  $\bar{s}_1^I = \hat{s}_1^I$ , which gives  $\Delta_2 = \Delta_4 = 0$  and  $\Delta_5^2 = \Phi_1^2$ ,  $\Delta_3^2 = \Phi_2^2$ , and (21) simplifies to

$$\begin{aligned} P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^e) \Big|_{h_4, h_5} &= Q \left( -\sqrt{\frac{h_5 \Phi_1^2 + h_4 \Phi_2^2}{2N_0}} \right) \\ &= 1 - P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^c) \Big|_{h_4, h_5} \quad (22) \end{aligned}$$

where  $P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^c) \Big|_{h_4, h_5}$  is the CPEP of the classical CIOD, which will be calculated in the sequel. As seen from (22), the error probability approaches unity with increasing SNR, which is somehow expected since both relays erroneously detected  $s_1$  and forwarded this signal to  $D$ .

**ii)**  $P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^e)$ : Assuming that  $s_1 = s_1^R + js_1^I$  has been correctly detected at  $R_1$  ( $\hat{s}_1 = s_1$ ) and erroneously detected as  $\hat{s}_1 = \hat{s}_1^R + j\hat{s}_1^I$  at  $R_2$ , by setting  $\Delta_3 = \Delta_4 = 0$ ,  $\Delta_2^2 = \Phi_2^2$  and  $\Delta_5^2 = \Phi_1^2$  in (21) for this case, we obtain

$$P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^e) \Big|_{h_4, h_5} = Q \left( \frac{-h_5 \Phi_1^2 + h_4 \Phi_2^2}{2\sigma \sqrt{h_5 \Phi_1^2 + h_4 \Phi_2^2}} \right) \quad (23)$$

which has a similar structure as that of (12), and the corresponding UPEP can be calculated as

$$P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^e) \approx P(h_4 \Phi_2^2 < h_5 \Phi_1^2) = \frac{\Phi_1^2}{\Phi_1^2 + \Phi_2^2}. \quad (24)$$

**iii)**  $P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^c)$ : Assuming that both of the relays have been successfully detected  $s_1$ , i.e.,  $\hat{s}_1 = s_1$  and  $\bar{s}_1 = s_1$ , we have  $\Delta_3 = \Delta_5 = 0$ ,  $\Delta_4^2 = \Phi_1^2$  and  $\Delta_2^2 = \Phi_2^2$ , and (21) simplifies to the CPEP of the classical CIOD as

$$P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^c) \Big|_{h_4, h_5} = Q \left( \sqrt{\frac{h_5 \Phi_1^2 + h_4 \Phi_2^2}{2N_0}} \right) \quad (25)$$

for which the UPEP can be calculated as follows:

$$P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^c) = \frac{1}{\pi} \int_0^{\pi/2} M_v \left( \frac{-1}{4N_0 \sin^2 \theta} \right) d\theta \quad (26)$$

where  $v \triangleq h_5 \Phi_1^2 + h_4 \Phi_2^2$  with

$$\begin{aligned} f_v(v) &= \frac{e^{-v/\Phi_1^2} - e^{-v/\Phi_2^2}}{\Phi_1^2 - \Phi_2^2}, \\ M_v(s) &= \frac{1}{(\Phi_1^2 s - 1)(\Phi_2^2 s - 1)}. \quad (27) \end{aligned}$$

Substituting (27) into (26), the desired UPEP, which is also the UPEP of the classical CIOD, can be calculated as

$$P_D(s_1 \rightarrow \hat{s}_1 | R_1^c, R_2^c) = \frac{\Phi_1^2(1 - \rho_1) + \Phi_2^2(-1 + \rho_2)}{2(\Phi_1^2 - \Phi_2^2)} \quad (28)$$

where  $\rho_1 \triangleq 1/\sqrt{1 + 4N_0/\Phi_1^2}$  and  $\rho_2 \triangleq 1/\sqrt{1 + 4N_0/\Phi_2^2}$ .

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