Outage Probability Analysis of Cooperative Spatial Modulation Systems

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***Abstract*—The capacity analysis of the spatial modulation (SM) scheme is different than that of the multiple-input multiple-output (MIMO) systems. Since the information is conveyed not only through the *M*-ary constellation domain but also through the antenna domain, the capacity of SM is expressed as the sum of the capacities of these two domains. Due to this summation, the outage probability of SM differs from the conventional systems. A detailed analysis of the outage probability of SM has not been given in the literature yet. In this work, first, we derive the outage probability performance of the classical SM system and second, we extend the results to the cooperative scenarios under fixed, selective and incremental relaying techniques. It is shown that SM and cooperative SM systems provide better performance compared to conventional modulation systems in terms of outage probability.**

I. Introduction

The demand for higher data rates and improved error performance lead the researchers to seek new and efficient transmission techniques. Multiple-input multiple-output (MIMO) transmission technologies cover this demand in the sense of increased channel capacity and/or improved error performance. Spatial modulation (SM) is a new and promising approach for MIMO systems where the information bits are carried through both the antenna indices and the conventional two dimensional signal constellations [1]. An SM symbol carries $\log_2(N_t M)$ information bits where N_t is the number of transmit antennas and M is the constellation size for the conventional amplitude/phase modulations (APM). The first $\log_2(N_t)$ bits are mapped into the transmit antenna index while the remaning bits are mapped into APM. Since a single antenna is activated during the transmission, interchannel interference (ICI) is completely eliminated in SM systems. A special case of SM, called space shift keying (SSK), carries information by only antenna indexes by eliminating APM [2]. Although the transceiver of SSK is simple and has less decoding complexity, its spectral efficiency is considerably lower compared to SM for the same number of transmit antennas.

Since SM scheme exploits the spatial domain for data transmission, its capacity calculation is different than the classical systems. The first study for the evaluation of the SM capacity is given in [3]. In this study, the mutual information of the space and the APM domains are calculated separately to obtain the total capacity for the SM scheme. In [4], this work is extended to SM with finite alphabet inputs for multiple input single output (MISO) channels. A comprehensive analysis is given in [5] where the mutual information of the maximum likelihood (ML) transmit antenna detector for the space domain is analyzed and SM-MISO capacities are given for complex Gaussian, real Gaussian and constant modulus APM distributions. Additionally, a lower bound is computed for SM-MIMO capacity for the complex Gaussian APM distribution.

When the signal power drops below a certain threshold (i.e., an outage event occurs), it is highly likely to have decoding failure. In that sense, the outage probability is an important performance measure for communication systems operating over fading channels. In this work, we investigate the outage probability of an SM system. To the best of our knowledge, a closed form expression for the outage probability of an SM system has not given in the literature yet. An outage probability analysis for the SM system based on transmit antenna selection can be found in [6]. However, the capacity of the SM system is assumed the same as conventional SIMO capacity. In [7], the outage probability of SM is given for the transmit antenna selection only and considering high SNR values. After carrying out an outage analysis of an SM system, we extend it to cooperative SM systems with different kind of diversity protocols which has not been studied before.

Cooperative communication has attracted numerous researchers over the past decade. In a cooperative senario, a source (S) transmits its data to the relay (R) and the destination (D) in the first phase and the relay forwards the source's information either decoding the received signal (decode-and-forward, DF) or amplifying it (amplify-and-forward, AF) in the second phase. This forwarding concept forms a virtual MIMO system to combat fading and it is very effective to gain a larger coverage. Besides the relay processing techniques, different kind of diversity protocols can be employed for a cooperative senario [8]. In selection relaying (SR), S sends its information to R and D in the first phase. In the second phase, if the instantaneous SNR between S and R falls below a certain threshold, S continues to transmit to D, if it is above the threshold, R forwards what it received using either DF or AF techniques. The spectral efficiency for fixed AF/DF relaying and SR is halved since the transmission takes two time slots. This is improved by incremental relaying (IR). In IR, if the S-D link SNR is not high enough for the appropriate communication, the relay participates the communication and transmits the received signal. Otherwise, R remains silent.

Since the information is conveyed by both antenna and APM domains, the capacity of an SM system will be the

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sum of the capacities of these two domains. Hence, the outage probability analysis of the SM scheme will not be the same as the conventional systems. In this work, the outage probability analysis of an SM system, which is not given in the literature, is investigated. The above analysis is extended to a cooperative system under different relaying techniques and different diversity protocols. It is shown that, the outage probability of the SM (cooperative SM) scheme provides better performance than conventional modulation techniques.

The rest of the paper is organized as follows. In Section II, the system model and the capacity bounds are given. The outage probability analysis for the SM system and extension to the cooperative scenario are investigated in two subsections. The numerical results and computer simulations are presented in Section III. Section IV concludes the paper.

Notation: A scalar, a vector and a matrix is denoted by a lower/upper-case italic, a lower-case boldface and an upper-case boldface letter, respectively. $(\cdot)^T$, $(\cdot)^H$ and $\|\cdot\|$ represents transpose, Hermitian transpose and Euclidean/Frobenius norm of a vector/matrix, respectively. We use $\mathbb{C}^{m\times n}$ to represent the dimensions of a complex valued matrix. $\Pr\{\cdot\}$ denotes the probability of an event, $I(X;Y)$ represents the mutual information between X and Y and $H(\cdot)$ is used for the entropy function. The probability density function (pdf) and the cumulative distribution function (cdf) of a random variable are given as $f_X(x)$, $F_X(x)$, respectively. $\mathcal{CN}(0,\sigma^2)$ denotes the circularly symmetrical zero-mean complex Gaussian distribution with variance σ^2 and $\mathbf{I}_m$ is the $m\times m$ identity matrix.

II. System Model

Consider a MIMO system consisting of N_t transmit and N_r receive antennas. A unit energy, $E[\mathbf{x}^H\mathbf{x}] = 1$, SM symbol can be regarded as $\mathbf{x} = [\underbrace{0,\ 0,\ \ldots,\ 0,}_{l-1}\ x_q,\ \underbrace{0,\ \ldots,\ 0}_{N_t-l}]^T$, where l is the active antenna index and x_q is the APM constellation symbol. The received signal is given as

$$\mathbf{y} = \mathbf{h}_l x_q + \mathbf{n} \tag{1}$$

where $\mathbf{h}_l \in \mathbb{C}^{N_r} \times 1$ vector is the l^{th} column of the MIMO channel matrix, $\mathbf{H} = [\ \mathbf{h}_1\ \ \mathbf{h}_2\ \ \ldots\ \ \mathbf{h}_{N_t}\ ]$, with each element being independent and identically distributed (i.i.d) with $\mathcal{CN}(0,1)$ and $\mathbf{n}$ is the $N_r \times 1$ additive white Gaussian vector whose entries are modeled as $\mathcal{CN}(0,N_0)$ with noise spectral density $N_0/2$ per dimension. The detector having ideal channel state information (CSI) estimates antenna index, l, and the APM signal, x_q, with the ML decision rule as follows:

$$[\hat{l},\hat{x}_q] = \arg\min_{l,q} \|\mathbf{y} - \mathbf{h}_l x_q\|^2 . \tag{2}$$

A. SM Capacity and Outage Probability

Since the information is conveyed not only by classical APM symbols, but also by the space domain in SM systems, the SM capacity is computed as the capacity of conventional modulation plus the capacity of the space domain [3],

$$I(X_q, X_{ch}; Y) = I(X_{ch}; Y) + I(X_q; Y \mid X_{ch}) \tag{3}$$

where X_q and X_{ch} represents APM and antenna domain symbol spaces, respectively and Y is the output symbol space. If the space domain symbols are considered equiprobable for flat fading channel, the mutual information between APM symbol X_q and the output symbol Y conditioned on the antenna domain symbol X_{ch} is given as,

$$\begin{aligned} I(X_q;Y \mid X_{ch}) &= H(Y \mid X_{ch}) - H(Y \mid X_q, X_{ch}) \\ &= \frac{1}{N_t}\sum_{l=1}^{N_t} \log_2(1+\rho\,\|\mathbf{h}_l\|^2) \end{aligned} \tag{4}$$

where ρ is the average signal to noise ratio (SNR) and the input symbols are considered as i.i.d. complex Gaussian ensembles. The first term in the right hand side of (3) can be calculated with the well-known differential entropy equation as,

$$I(X_{ch};Y) = H(Y) - H(Y \mid X_{ch}). \tag{5}$$

Generally, a closed form expression for (5) is difficult to obtain and solution can be calculated numerically. However, when no antenna index error occurs at the receiver, the information carried by space domain is transferred fully and (5) becomes $I(X_{ch};Y) = \log_2(N_t)$. Hence, the SM capacity can be bounded as [7]

$$\begin{aligned} \frac{1}{N_t}\sum_{l=1}^{N_t} \log_2\left(1+\rho\,\|\mathbf{h}_l\|^2\right) &\le C_{SM} \\ &\le \frac{1}{N_t}\sum_{l=1}^{N_t} \log_2\left(1+\rho\,\|\mathbf{h}_l\|^2\right) + \log_2\left(N_t\right). \end{aligned} \tag{6}$$

In (6), the right hand side is the full SM capacity with no antenna index error.

SM capacity in (3) averages the mutual information of each link from all transmit antennas. However, since only one antenna is active for a given time interval, the outage event occurs over the activated link. Hence, the instantaneous capacity have to be found for the outage probability analysis.

Consider the set of N_t channel vector norms, $\left\{\|\mathbf{h}_l\|^2\right\}_{l=1}^{N_t}$, an outage event occurs when $I(X_q, X_{ch=\min};Y) \le R$. Here, R is the pre-determined rate in bits/s/Hz and $X_{ch=\min}$ represents the space domain symbols when the lowest vector norm, $\|\mathbf{h}_{\min}\|^2$, is chosen. In this way, (4) and (5) respectively simplify to $\log_2\left(1+\rho\,\|\mathbf{h}_{\min}\|^2\right)$ and $H(Y) - H(Y \mid X_{ch=\min})$. Therefore, the instantenous mutual information for the $l = \min$ link is

$$\begin{aligned} I(X_q, X_{ch=\min};Y) &= \log_2\left(1+\rho\,\|\mathbf{h}_{\min}\|^2\right) \\ &\quad + H(Y) - H(Y \mid X_{ch=\min}). \end{aligned} \tag{7}$$

Here, entropies can be written as,

$$H(Y) = -\int_{\mathbf{y}} f_Y(\mathbf{y}) \log_2 f_Y(\mathbf{y}) d\mathbf{y} \tag{8}$$

$$\begin{aligned} H(Y \mid X_{ch=\min}) &= -\int_{\mathbf{y}} f_Y(\mathbf{y} \mid x_{ch=\min}) \\ &\qquad \times \log_2 f_Y(\mathbf{y} \mid x_{ch=\min}) d\mathbf{y} \\ &= \log_2 \det\left(\pi e \mathbf{K}_{\min}\right) \end{aligned} \tag{9}$$

where $f_Y(\mathbf{y}) = \frac{1}{N_t}\sum_{l=1}^{N_t}\frac{1}{\det(\pi\mathbf{K}_l)}\exp\left(-\mathbf{y}^H\mathbf{K}_l^{-1}\mathbf{y}\right)$ and $\mathbf{K}_l = \mathbf{h}_l\mathbf{h}_l^H\sigma_x^2 + N_0\mathbf{I}_{N_r}$. Besides, $f_Y(\mathbf{y} \mid x_{ch=\min}) = \frac{1}{\det(\pi\mathbf{K}_{\min})}\exp\left(-\mathbf{y}^H\mathbf{K}_{\min}^{-1}\mathbf{y}\right)$ and $\mathbf{K}_{\min} = \mathbf{h}_{\min}\mathbf{h}_{\min}^H\sigma_x^2 + N_0\mathbf{I}_{N_r}$ where σ_x^2 is the signal power. (8) can be computed with numerical integration. On the other hand, since (8) and (10) depend on the space domain symbols which are in fact the channel gains, $H(Y) - H(Y \mid X_{ch=\min})$ have to be averaged. After computing the $H(Y) - H(Y \mid X_{ch=\min})$, the exact outage probability is

$$\begin{aligned} P_{out}^{SM} &= \Pr\left\{I(X_q, X_{ch=\min}; Y) \leq R\right\} \quad (10)\\ &= \Pr\left\{\|\mathbf{h}_{\min}\|^2 \leq \frac{2^{R-H(Y)+H(Y|X_{ch=\min})}-1}{\rho}\right\}. \end{aligned}$$

The pdf of the $\nu \triangleq \|\mathbf{h}_{\min}\|^2$ which is the minimum of the channel vector norms, $\ell \triangleq \mathbf{h}_l$, $l = 1, \ldots, N_t$, (Erlang random variables) can be calculated with the help of order statistics as

$$f_\nu(x) = N_t\left(1 - F_\ell(x)\right)^{N_t-1} f_\ell(x) \quad (11)$$

where the $F_\ell(x)$ and $f_\ell(x)$ denote the cdf and the pdf of the channel vector norms, respectively. Hence,

$$\begin{aligned} f_\nu(x) &= N_t\left(e^{-x}\sum_{k=0}^{N_r-1}\frac{x^k}{k!}\right)^{N_t-1}\frac{x^{N_r-1}e^{-x}}{(N_r-1)!}\\ &= \frac{N_t e^{-N_t x}}{(N_r-1)!}\sum_{k=0}^{(N_r-1)(N_t-1)} C(k)x^{N_r+k-1} \quad (12) \end{aligned}$$

where $C(k)$ is the coefficient of x^k in the expansion of $\left(\sum_{k=0}^{N_r-1}\frac{x^k}{k!}\right)^{N_t-1}$. Therefore, the outage probability for SM systems is given as

$$\begin{aligned} P_{out}^{SM} &= \int_0^{\rho^{th}}\frac{N_t e^{-N_t x}}{(N_r-1)!}\sum_{k=0}^{N} C(k)x^{N_r+k-1}\\ &= \sum_{k=0}^{N}\frac{C(k)\gamma(N_r+k, N_t\rho^{th})}{N_t^{N_r+k-1}(N_r-1)!}. \quad (13) \end{aligned}$$

In (13), $\rho^{th} = \frac{2^{R-H(Y)+H(Y|X_{ch=\min})}-1}{\rho}$, $\gamma(\cdot,\cdot)$ represents lower incomplete Gamma function and $N = (N_r-1)(N_t-1)$.

Furthermore, the lower and upper bounds for outage probability of SM can be written as

$$\begin{aligned} &\Pr\left\{\log_2\left(1+\rho\|\mathbf{h}_{\min}\|^2\right) \leq R\right\} \geq P_{out}\\ &\geq \Pr\left\{\log_2\left(1+\rho\|\mathbf{h}_{\min}\|^2\right) + \log_2(N_t) \leq R\right\}. \quad (14) \end{aligned}$$

B. Outage Probability of Cooperative SM Systems

In a cooperative system, consisting of a source (S), a relay (R) and a destination (D), S and R have N_t^S, and N_t^R transmit antennas, while R and D have N_r^R and N_r^D receive antennas, respectively.

Fading coefficients of each link of the MIMO channels between S and R, $\mathbf{H}^{SR} \in \mathbb{C}^{N_r^R \times N_t^S}$, S and D, $\mathbf{H}^{SD} \in \mathbb{C}^{N_r^D \times N_t^S}$, R and D, $\mathbf{H}^{RD} \in \mathbb{C}^{N_r^D \times N_t^R}$, can be modeled as $\mathcal{CN}(0,1)$ and obey the flat Rayleigh fading channel conditions. In the first time slot, S transmits an SM symbol to D and R as,

$$\mathbf{y}^{SD} = \mathbf{h}_l^{SD}x_q + \mathbf{n}^{SD} \quad (15)$$

$$\mathbf{y}^{SR} = \mathbf{h}_l^{SR}x_q + \mathbf{n}^{SR}, \quad (16)$$

respectively, where $\mathbf{h}_l^{SD(SR)}$ denotes the l^{th} column vector of the channel matrix $\mathbf{H}^{SD(SR)}$. Based on the DF relaying protocol, R processes the incoming data using the ML detection and makes a decision according to

$$[\tilde{l}, \tilde{x}_q] = \arg\min_{l,q}\left\|\mathbf{y}^{SR} - \mathbf{h}_l^{SR}x_q\right\|^2 \quad (17)$$

and re-encodes to an SM signal and sends to D as

$$\mathbf{y}_{DF}^{RD} = \mathbf{h}_{\tilde{l}}^{RD}\tilde{x}_q + \mathbf{n}^{RD}. \quad (18)$$

On the other hand, in AF relaying protocol, R just amplifies the received signal and forwards to D as

$$\mathbf{y}_{AF}^{RD} = G\mathbf{H}^{RD}\mathbf{y}^{SR} + \mathbf{n}^{RD} \quad (19)$$

where $G = \sqrt{\frac{1}{\left\|\mathbf{h}_l^{SR}\right\|^2 + N_r^R N_0}}$ is the gain factor which is chosen to fix the transmitted energy of the relay at unity. $\mathbf{n}^{SD}, \mathbf{n}^{SR}$ and $\mathbf{n}^{RD}$ are the additive Gaussian noise vectors whose entries are distributed with $\mathcal{CN}(0, N_0)$. After simple manipulations, (19) can be written as

$$\begin{aligned} \mathbf{y}_{AF}^{RD} &= G\mathbf{H}^{RD}\left(\mathbf{h}_l^{SR}x_q + \mathbf{n}^{SR}\right) + \mathbf{n}^{RD}\\ &= G\mathbf{H}^{RD}\mathbf{h}_l^{SR}x_q + \tilde{\mathbf{n}}^{RD} \quad (20) \end{aligned}$$

where $\tilde{\mathbf{n}}^{RD}$ is the effective noise vector with a covariance matrix as $\mathbf{C} = G^2\mathbf{H}^{RD}\left(\mathbf{H}^{RD}\right)^H N_0 + N_0\mathbf{I}_{N_r^D}$.

The outage probability analysis for classical modulation techniques and different relay and diversity protocols can be found in [8]. In this study, we have used the results for these protocols and applied them to the cooperative SM systems. In the following, the antenna numbers are assumed to be equal, i.e., $N_t^S = N_t^R = N_t$ and $N_r^R = N_r^D = N_r$.

1) Fixed-DF Relaying: The mutual information of fixed DF-SM is given as

$$\begin{aligned} I_{DF_{SM}} = \min\Big\{&\log_2\left(1+\rho\left\|\mathbf{h}_{\min}^{SR}\right\|^2\right)\\ &+ H(Y^{SR}) - H(Y^{SR} \mid X_{ch=\min}),\\ &\log_2\left(1+\rho\left\|\mathbf{h}_{\min}^{SD}\right\|^2 + \left\|\mathbf{h}_{\min}^{RD}\right\|^2\right) \quad (21)\\ &+H(Y^{SD+RD}) - H(Y^{SD+RD} \mid X_{ch=\min})\Big\} \end{aligned}$$

and the corresponding outage probability is

$$\begin{aligned} P_{out}^{DF_{SM}} = &\Pr\left\{\left\|\mathbf{h}_{\min}^{SR}\right\|^2 \leq \mu\right\} + \Pr\left\{\left\|\mathbf{h}_{\min}^{SR}\right\|^2 > \mu\right\}\\ &\times \Pr\left\{\left\|\mathbf{h}_{\min}^{SD}\right\|^2 + \left\|\mathbf{h}_{\min}^{RD}\right\|^2 \leq \mu\right\} \quad (22) \end{aligned}$$

where $H(Y^{SR})$ and $H(Y^{SD+RD})$ can be computed as in (8) with variances $\sigma_{SR}^2 = \left\|\mathbf{h}_l^{SR}\right\|^2\sigma_x^2 + N_0$ and $\sigma_{SD+RD}^2 = \left(\left\|\mathbf{h}_l^{SD}\right\|^2 + \left\|\mathbf{h}_l^{RD}\right\|^2\right)\sigma_x^2 + N_0$. On the other hand, $H(Y^{SR} \mid X_{ch=\min})$ and $H(Y^{SD+RD} \mid$

$X_{ch=\min})$ can be computed as in (10) with the above variances when $\mathbf{h}_l = \mathbf{h}_{\min}$. The threshold value is $\mu = \frac{1}{\rho}\left(2^{2R-H(Y^{SR/SD+RD})+H(Y^{SR/SD+RD}|X_{ch=\min})} - 1\right)$. As seen from the classical SM systems (see. Fig. 1), $H(Y) - H(Y \mid X_{ch=\min})$ equals to $\log_2(N_t)$ for mid to high SNR values. For cooperative communications, this value converges to $\log_2(N_t)$ faster than the classical systems. In this work, $H(Y) - H(Y \mid X_{ch=\min})$ is fixed to $\log_2(N_t)$ thus, the threshold value will be $\mu = \frac{2^{2R-\log_2(N_t)} - 1}{\rho}$. In (22), the first term in the right hand side can be calculated from (13). The second term is the complement of the first term, i.e., $\Pr\left\{\left\|\mathbf{h}_{\min}^{SR}\right\|^2 \leq \mu\right\}$. Finally, the last term can be evaluated using the pdf of sum of two independent random variables which is the convolution of their pdfs. If the pdf of each term is given as in (12), the pdf of their sum is calculated as

$$\begin{aligned} f_Z(z) &= \int_0^z f_{\mathcal{H}}(z-x) f_{\mathcal{H}}(x) dx \\ &= \frac{N_t^2 e^{-N_t z}}{(N_r-1)!^2} \sum_{k=0}^{N} \sum_{n=0}^{N} C(k) C(n) z^{2N_r+k+n-1} \\ &\qquad \times B(N_r+k, N_r+n) \end{aligned} \quad (23)$$

where $B(.,.)$ is the Beta function. The outage probability is then expressed as,

$$\begin{aligned} \Pr\left\{\left\|\mathbf{h}_{\min}^{SD}\right\|^2 + \left\|\mathbf{h}_{\min}^{RD}\right\|^2 \leq \mu^{th}\right\} &= \int_0^{\mu^{th}} f_Z(z) dz. \\ = \sum_{k=0}^{N} \sum_{n=0}^{N} & \frac{C(k)C(n)B(N_r+k, N_r+n)}{(N_r-1)!^2 N_t^{2N_r+k+n-2}} \\ & \times \gamma\left(2N_r+k+n, N_t \mu^{th}\right). \end{aligned} \quad (24)$$

2) DF Selection Relaying: The outage probability for DF-SR is given as

$$\begin{aligned} P_{out}^{DF-SR_{SM}} &= \Pr\left\{\left\|\mathbf{h}_{\min}^{SR}\right\|^2 \leq \mu\right\} \Pr\left\{2\left\|\mathbf{h}_{\min}^{SD}\right\|^2 \leq \mu\right\} \\ &+ \Pr\left\{\left\|\mathbf{h}_{\min}^{SR}\right\|^2 > \mu\right\} \Pr\left\{\left\|\mathbf{h}_{\min}^{SD}\right\|^2 + \left\|\mathbf{h}_{\min}^{RD}\right\|^2 \leq \mu\right\} \end{aligned} \quad (25)$$

3) DF Incremental Relaying: In incremental relaying, if the S-D link SNR is not high enough for the appropriate communication, the relay participates in the communication and transmits the received signal. Otherwise, R remains silent (i.e., there is no second phase). Since the relay does not cooperate in every time slot, the spectral efficiency will be R when relay is silent and $R/2$ otherwise. Therefore, the averaged spectral efficiency can be expressed as

$$\bar{R}_l = R \Pr\left\{\left\|\mathbf{h}_{\min}^{SD}\right\|^2 > \mu'\right\} + \frac{R}{2} \Pr\left\{\left\|\mathbf{h}_{\min}^{SD}\right\|^2 \leq \mu'\right\} \quad (26)$$

where $\mu' = \frac{2^{R-log_2(N_t)}-1}{\rho}$. (26) can be evaluated using (13). The outage probability of DF-IR is written as,

$$\begin{aligned} P_{out}^{DF-IR_{SM}} &= \Pr\left\{\left\|\mathbf{h}_{\min}^{SR}\right\|^2 \leq \bar{\mu}\right\} \Pr\left\{\left\|\mathbf{h}_{\min}^{SD}\right\|^2 \leq \bar{\mu}\right\} \\ &+ \Pr\left\{\left\|\mathbf{h}_{\min}^{SR}\right\|^2 > \bar{\mu}\right\} \Pr\left\{\left\|\mathbf{h}_{\min}^{SD}\right\|^2 + \left\|\mathbf{h}_{\min}^{RD}\right\|^2 \leq \bar{\mu}\right\} \end{aligned} \quad (27)$$

where $\bar{\mu} = \frac{2^{\bar{R}-log_2(N_t)}-1}{\rho}$.

4) Fixed AF Relaying: The outage probability for fixed AF relaying can be written as a function of fading coefficients as (The analysis for AF relaying is only given for $N_r = 1$)

$$P_{out}^{AF_{SM}} = \Pr\left\{\left|h_{\min}^{SD}\right|^2 + \frac{1}{\rho} f\left(\rho\left|h_{\min}^{SR}\right|^2, \rho\left|h_{\min}^{RD}\right|^2\right) \leq \mu\right\} \quad (28)$$

where $f(x,y) = \frac{xy}{x+y+1}$ [8]. In order to find the pdf of the left hand side of the inequality, we can employ the upper bound approach as performed in [9]. The expressed function can be upper bounded as,

$$f(x,y) = \frac{xy}{x+y+1} \leq \min(x,y). \quad (29)$$

Hence, the outage probability is re-expressed as

$$P_{out}^{AF_{SM}} \approx \Pr\left\{\left|h_{\min}^{SD}\right|^2 + \min\left(\left|h_{\min}^{SR}\right|^2, \left|h_{\min}^{RD}\right|^2\right) \leq \mu\right\} \quad (30)$$

For $N_r = 1$, each $\left\{|h_l|^2\right\}_{l=1}^{N_t}$ is exponentially distributed. Hence $\xi \triangleq |h_{\min}|^2$ is also exponentially distributed with the pdf $f_{\Xi}(\xi) = N_t e^{-N_t \xi}$ and the cdf $F_{\Xi}(\xi) = 1 - e^{-N_t \xi}$. Therefore the pdf of $\omega = a + \min(b,c)$ is $f_{\Omega}(\omega) = 2N_t\left(e^{-N_t \omega} - e^{-2N_t \omega}\right)$. Finally, the outage probability of AF relaying SM is given as

$$P_{out}^{AF} \approx \int_0^{\mu} f_{\Omega}(\omega) dz = 1 - 2e^{-N_t \mu} + e^{-2N_t \mu}. \quad (31)$$

5) AF Incremental Relaying: The outage probability of AF-IR can be written as,

$$\begin{aligned} P_{out}^{AF-IR_{SM}} &= \Pr\left\{\left|h_{\min}^{SD}\right|^2 \leq \bar{\mu}\right\} \Pr\left\{\left|h_{\min}^{SD}\right|^2 \right. \\ &\quad \left. + \min\left(\left|h_{\min}^{SR}\right|^2, \left|h_{\min}^{RD}\right|^2\right) \leq \bar{\mu} \,\middle|\, \left|h_{\min}^{SD}\right|^2 \leq \bar{\mu}\right\} \\ &= \Pr\left\{\left|h_{\min}^{SD}\right|^2 + \min\left(\left|h_{\min}^{SR}\right|^2, \left|h_{\min}^{RD}\right|^2\right) \leq \bar{\mu}\right\}. \end{aligned} \quad (32)$$

The analytical expression for (32) is the same as in (31), but the only difference is μ replaced by $\bar{\mu}$.

III. Performance Evaluation

In this section, we present computer simulation results for the outage probabilities of SM and cooperative SM systems. Monte Carlo simulations are realized for at least 10^7 channel uses as a function of received SNR (ρ) and obtained curves are compared with the analytical results. On the other hand, all SM systems are compared with the conventional modulation techniques on an equal spectral efficiency basis.

A. Results for SM system

SM with $R = 3$ and $R = 4$ bits/s/Hz and corresponding *M*-PSK systems are compared in Fig. 1. The outage probabilities of SM systems with $N_t = 4$, BPSK and $N_t = 4$, QPSK are simulated. The upper and lower bounds are also depicted for comparison purposes. Additionally, they are compared with 8-PSK and 16-PSK systems. As seen from Fig. 1, the exact outage probability lies between the upper and lower bounds, as expected. At low SNR values, it is highly probable that the receiver makes antenna index error which makes the exact

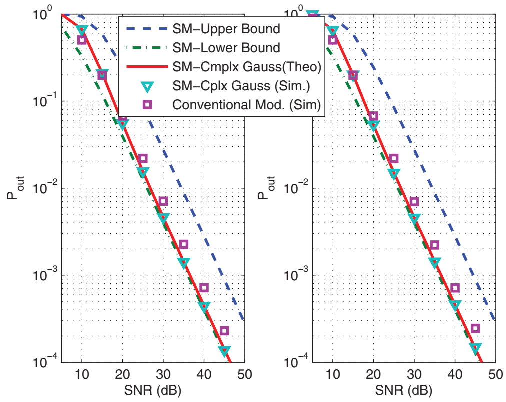

Fig. 1: Outage probability performance of $N_t = 4$, BPSK (left) and $N_t = 4$, QPSK (right) SM systems compared with SISO 8-PSK and 16-PSK ($N_r = 1$).

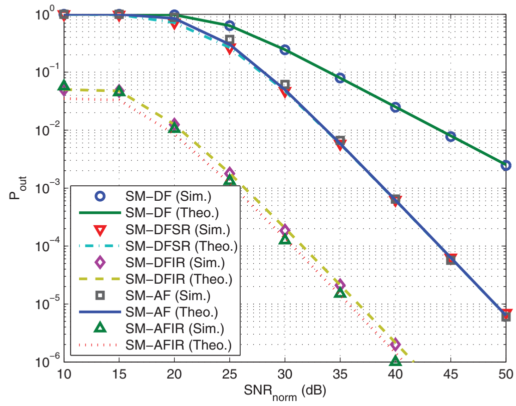

Fig. 2: Outage probability performance of $N_t = 4$ QPSK AF vs. DF Cooperative SM system ($N_r = 1$).

curve closer to upper bound. At high SNR values, it is less likely to have antenna index error and the outage probability converges to lower bound. Also note that the analytical curves obtained from (13) exactly match with the simulation results. The outage probability performance of SM system gets better at high SNR values as expected. On the other hand, since we selected the minimum channel gains for computing the outage probability, this performance is the worst case senario. Hence, the better outage probability performance can be expected in practice.

B. Results for Cooperative SM

The outage probability performance of fixed DF, DF-SR, DF-IR and fixed AF, AF-IR of an $N_t = 4$, QPSK SM system is presented in Fig. 2. As seen from Fig. 2, the simulation results and the analytical curves have exact match. In [8] and [9], it is stated that the AF relaying is superior to DF relaying in terms of outage probability. This result is observed in here as well. The performance of cooperative SM and the corresponding PSK systems is presented in Fig. 3 for $R = 4$ bits/s/Hz. As seen from Fig. 3, the cooperative SM system provides better performance than corresponding conventional modulated systems for both DF and AF protocols.

IV. Conclusion

In this paper, we have evaluated the outage probability performance of both classical SM and cooperative SM systems. Due to the fact that SM conveys information not only by the modulated symbols, but also by the antenna indices, the capacity calculation of SM differs from the conventional systems. The corresponding capacity can be calculated as the sum of the capacities of APM and the space domains. Related to the capacity, the outage probability performance of an SM system, which is not thoroughly examined in the literature, is deeply affected by the antenna index errors. In this work, we have derived the outage probability performance of point to point SM and the cooperative SM systems for the worst case senario. In both systems, although the minimum channel gains are selected, a better performance is achieved compared to the classical systems.

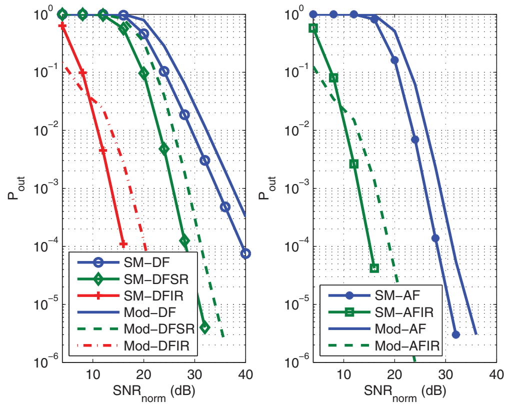

Fig. 3: Outage probability performance of $N_t = 4$ QPSK Co-operative SM system with respect to SIMO 16-PSK ($N_r = 2$).

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