

# Cognitive cross network design with physical-layer coding and spatial modulation

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A cognitive cross network where both primary and secondary users' receivers are closer to the transmitter of the other user; however, the distances to their own transmitters are too large that a relay is required for a reliable communication, is considered. To this end, the secondary user shares its relay with the primary user (PU) by means of physical-layer network coding in exchange for access to PU's licensed band. It is assumed that both transmitters and the relay adopt the emerging spatial modulation concept to improve the error performance. Bit error probability of the proposed scheme is analytically derived and supported via computer simulation results.

**Introduction:** Cognitive radio (CR), which allows secondary (unlicensed) users (SUs) to access the frequency bands of primary (licensed) users (PUs), is a solution to scarcity and inefficient use of the available spectrum [1, 2]. Spatial modulation (SM) that conveys information by antenna indices besides  $M$ -ary modulation schemes, is a multiple-input multiple-output transmission technique employing a single RF chain at the transmitter [3, 4]. It provides a perfect compromise between spectral efficiency and energy efficiency as well as reliability. In bidirectional transmission over a relay, the spectral efficiency can also be improved by physical-layer network coding (PLNC) where the relay receives signals of two users in multiple access (MA) phase and applies bit-wise exclusive-or (XOR) operation to combine these signals, then brings out the PLNC mapped signal in broadcast (BC) phase [5]. Therefore, the number of required time intervals is reduced to two and the spectral efficiency is increased.

In this Letter, a cross network combining CR with SM and PLNC techniques is considered where each user's receiver is closer to the transmitter of the other user and the distances between both user's transmitter-receiver pairs are too large that a relay is required for a reliable communication. SU consisting of a secondary transmitter (ST) and a secondary receiver (SR), shares its relay (R) with PU having a primary transmitter (PT) and a primary receiver (PR). In return, SU can access PU's licensed spectrum and perform its own transmission. PT, ST and R adopt SM while R also applies PLNC. Bit error probability (BEP) of the proposed scheme is analytically derived and the theoretical results are supported via computer simulations that show the provided improvements compared with the corresponding reference schemes.

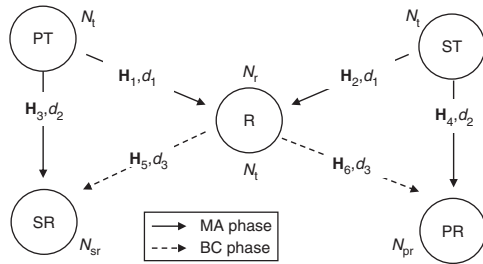


Fig. 1 Proposed spatially modulated cognitive cross network

**Notation:**  $(\cdot)^H$  denotes Hermitian transpose.  $E[X] = m_X$  and  $\text{Var}[X] = \sigma_X^2$  stand for the expected value and the variance of a random variable  $X$ , respectively. Bold-uppercase letters denote matrices while bold-lowercase letters represent vectors.  $x_i = (k_i, z_i)$  stands for the SM symbol, where  $k_i$  denotes the activated antenna index carrying  $\log_2(N_i)$  bits while  $z_i$  represents  $M$ -PSK symbol for  $i \in \{p, s, r\}$ .

**System model:** The considered cognitive cross network is given in Fig. 1 where  $N_t$  denotes the number of transmit antennas at PT, ST and R whereas  $N_{pr}$ ,  $N_{sr}$  and  $N_r$  represent the number of receive antennas at PR, SR and R, respectively.  $d_m$ ,  $m = 1, 2, 3$  are the distances between nodes.  $\mathbf{H}_j$ ,  $j = 1, 2, \dots, 6$  represent the matrices of channel fading coefficients whose components are assumed to be zero mean complex Gaussian random variables with variance  $d_m^{-\nu}$ , where  $\nu$  is the path-loss exponent. All noise components are assumed to be samples of additive white Gaussian noise (AWGN) process with variance  $N_0$ .  $P_p$ ,  $P_s$  and  $P_r$  denote the transmission powers of PT, ST and R, respectively.

In MA phase, PT and ST transmit simultaneously an SM symbol  $x_i = (k_i, z_i)$  to R where  $i = p$  and  $i = s$  for PT and ST, respectively. These transmissions are eavesdropped by SR and PR, respectively. The received signals at R, SR and PR are given by

$$\mathbf{y}_R = \sqrt{P_p} \mathbf{h}_{1k_p} z_p + \sqrt{P_s} \mathbf{h}_{2k_s} z_s + \mathbf{n}_R \quad (1)$$

$$\mathbf{y}_{SR1} = \sqrt{P_p} \mathbf{h}_{3k_p} z_p + \mathbf{n}_{SR1} \quad (2)$$

$$\mathbf{y}_{PR1} = \sqrt{P_s} \mathbf{h}_{4k_s} z_s + \mathbf{n}_{PR1} \quad (3)$$

where  $\mathbf{h}_{1k_p}$  and  $\mathbf{h}_{3k_p}$  are the  $k_p^{\text{th}}$  columns of  $\mathbf{H}_1$  and  $\mathbf{H}_3$ ,  $\mathbf{h}_{2k_s}$  and  $\mathbf{h}_{4k_s}$  are the  $k_s^{\text{th}}$  columns of  $\mathbf{H}_2$  and  $\mathbf{H}_4$ , and  $\mathbf{n}_R$ ,  $\mathbf{n}_{SR1}$  and  $\mathbf{n}_{PR1}$  represent AWGN vectors at R, SR and PR, respectively.

R, SR and PR perform the maximum likelihood (ML) detection as

$$(\hat{k}_p^R, \hat{z}_p^R, \hat{k}_s^R, \hat{z}_s^R) = \arg \min_{k_p, z_p, k_s, z_s} \|\mathbf{y}_R - \sqrt{P_p} \mathbf{h}_{1k_p} z_p - \sqrt{P_s} \mathbf{h}_{2k_s} z_s\|^2 \quad (4)$$

$$(\hat{k}_p^{SR}, \hat{z}_p^{SR}) = \arg \min_{k_p, z_p} \|\mathbf{y}_{SR1} - \sqrt{P_p} \mathbf{h}_{3k_p} z_p\|^2 \quad (5)$$

$$(\hat{k}_s^{PR}, \hat{z}_s^{PR}) = \arg \min_{k_s, z_s} \|\mathbf{y}_{PR1} - \sqrt{P_s} \mathbf{h}_{4k_s} z_s\|^2 \quad (6)$$

respectively. In BC phase, R BCs the PLNC mapped symbol  $x_r = (k_r, z_r)$  which is the SM symbol obtained from XOR operations of binary corresponding of  $\hat{k}_p^R$  and  $\hat{k}_s^R$ , and  $\hat{z}_p^R$  and  $\hat{z}_s^R$ , respectively. The received signals at SR and PR are given as

$$\mathbf{y}_{SR2} = \sqrt{P_r} \mathbf{h}_{5k_r} z_r + \mathbf{n}_{SR2} \quad (7)$$

$$\mathbf{y}_{PR2} = \sqrt{P_r} \mathbf{h}_{6k_r} z_r + \mathbf{n}_{PR2} \quad (8)$$

where  $\mathbf{h}_{5k_r}$  and  $\mathbf{h}_{6k_r}$  are the  $k_r^{\text{th}}$  columns of  $\mathbf{H}_5$  and  $\mathbf{H}_6$ , and  $\mathbf{n}_{SR2}$  and  $\mathbf{n}_{PR2}$  stand for the AWGN vectors at SR and PR, respectively.

In BC phase, SR and PR detect the PLNC mapped symbol according to the ML detection rule as

$$(\hat{k}_r^{SR}, \hat{z}_r^{SR}) = \arg \min_{k_r, z_r} \|\mathbf{y}_{SR2} - \sqrt{P_r} \mathbf{h}_{5k_r} z_r\|^2 \quad (9)$$

$$(\hat{k}_r^{PR}, \hat{z}_r^{PR}) = \arg \min_{k_r, z_r} \|\mathbf{y}_{PR2} - \sqrt{P_r} \mathbf{h}_{6k_r} z_r\|^2 \quad (10)$$

respectively. SR and PR can obtain their target bits by applying XOR operation to the binary corresponding of the following pairs:  $(\hat{k}_p^{SR}, \hat{z}_p^{SR})$  and  $(\hat{k}_r^{SR}, \hat{z}_r^{SR})$  for SR, and  $(\hat{k}_s^{PR}, \hat{z}_s^{PR})$  and  $(\hat{k}_r^{PR}, \hat{z}_r^{PR})$  for PR.

**BEP analysis:** BEP of PU can be expressed as

$$P_b^{PR} \leq 1 - (1 - P_b^R)(1 - P_b^{ST \rightarrow PR})(1 - P_b^{R \rightarrow PR}) \quad (11)$$

where  $P_b^R$  stands for BEP of R, which consists of both PT  $\rightarrow$  R and ST  $\rightarrow$  R links, i.e. MA channel (MAC). In (11),  $P_b^{ST \rightarrow PR}$  and  $P_b^{R \rightarrow PR}$  denote BEPs of the ST  $\rightarrow$  PR and R  $\rightarrow$  PR links, respectively. When the SM symbols  $x_p$  and  $x_s$  are transmitted to R by PT and ST, the decision metric can be written from (4) as  $m(k_p, z_p, k_s, z_s) = \|\mathbf{y}_R - \sqrt{P_p} \mathbf{h}_{1k_p} z_p - \sqrt{P_s} \mathbf{h}_{2k_s} z_s\|^2$ . Then, the conditional pairwise error probability (CPEP) for MAC at R when  $x_p$  and  $x_s$  are transmitted and erroneously detected as  $\hat{x}_i^R = (\hat{k}_i^R, \hat{z}_i^R)$ ,  $i = \{p, s\}$ , is given by

$$\begin{aligned} \text{CPEP}^{\text{MAC}} &= P(m(k_p, z_p, k_s, z_s) \geq m(\hat{k}_p^R, \hat{z}_p^R, \hat{k}_s^R, \hat{z}_s^R)) \\ &= P(D \geq 0 | \mathbf{h}_{1k_p}, \mathbf{h}_{2k_s}) = Q\left(-\frac{m_D}{\sigma_D}\right) \end{aligned} \quad (12)$$

where the decision variable is  $D = -\|\mathbf{h}_{1k_p} z_p - \mathbf{h}_{1\hat{k}_p^R} \hat{z}_p^R + \mathbf{h}_{2k_s} z_s - \mathbf{h}_{2\hat{k}_s^R} \hat{z}_s^R\|^2 - 2\Re\{\mathbf{n}_R^H(\mathbf{h}_{1k_p} z_p - \mathbf{h}_{1\hat{k}_p^R} \hat{z}_p^R + \mathbf{h}_{2k_s} z_s - \mathbf{h}_{2\hat{k}_s^R} \hat{z}_s^R)\}$  with  $m_D = -\|\mathbf{h}_{1k_p} z_p - \mathbf{h}_{1\hat{k}_p^R} \hat{z}_p^R + \mathbf{h}_{2k_s} z_s - \mathbf{h}_{2\hat{k}_s^R} \hat{z}_s^R\|^2$  and  $\sigma_D^2 = 2N_0 \|\mathbf{h}_{1k_p} z_p - \mathbf{h}_{1\hat{k}_p^R} \hat{z}_p^R + \mathbf{h}_{2k_s} z_s - \mathbf{h}_{2\hat{k}_s^R} \hat{z}_s^R\|^2$ .

Using the alternative form of  $Q$  function, (12) can be rewritten as

$$\text{CPEP}^{\text{MAC}} = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{\|\mathbf{h}_{1k_p} z_p - \mathbf{h}_{1\hat{k}_p^R} \hat{z}_p^R + \mathbf{h}_{2k_s} z_s - \mathbf{h}_{2\hat{k}_s^R} \hat{z}_s^R\|^2}{4N_0 \sin^2 \theta}\right) d\theta \quad (13)$$

After taking the expectation of (13) over the vectors of channel fading coefficients, the average pairwise error probability (APEP) at R can be given as

$$\text{APEP}^{\text{MAC}} = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{\sin^2 \theta}{\sin^2 \theta + (c/4N_0)} \right)^{N_r} d\theta \quad (14)$$

$$= \frac{1}{2} \left[ 1 - \mu \left( \frac{c}{4N_0} \right) \sum_{m=0}^{N_r-1} \binom{2m}{m} \left( \frac{1 - \mu^2(c/4N_0)}{4} \right)^m \right] \quad (15)$$

where  $\mu(\ell) = \sqrt{\ell/(1+\ell)}$  and

$$c = \begin{cases} d_1^{-v} (|z_p - \hat{z}_p^R|^2 + |z_s - \hat{z}_s^R|^2), & k_p = \hat{k}_p^R \ \& \ k_s = \hat{k}_s^R \\ d_1^{-v} (|z_p|^2 + |\hat{z}_p^R|^2 + |z_s - \hat{z}_s^R|^2), & k_p \neq \hat{k}_p^R \ \& \ k_s = \hat{k}_s^R \\ d_1^{-v} (|z_p - \hat{z}_p^R|^2 + |z_s|^2 + |\hat{z}_s^R|^2), & k_p = \hat{k}_p^R \ \& \ k_s \neq \hat{k}_s^R \\ d_1^{-v} (|z_p|^2 + |\hat{z}_p^R|^2 + |z_s|^2 + |\hat{z}_s^R|^2), & k_p \neq \hat{k}_p^R \ \& \ k_s \neq \hat{k}_s^R \end{cases} \quad (16)$$

Finally, BEP of R can be expressed as

$$P_b^R = \frac{1}{M^2 N_t} \sum_{x_p} \sum_{x_s} \sum_{\hat{x}_p} \sum_{\hat{x}_s} \text{APEP}^{\text{MAC}} \times \frac{e(x_r \rightarrow \hat{x}_r)}{\log_2(MN_t)} \quad (17)$$

where  $e(x_r \rightarrow \hat{x}_r)$  is the number of erroneous bits in detection of the PLNC mapped symbol at R.

For  $P_b^{\text{R} \rightarrow \text{SR}}$ , the corresponding decision metric is given from (10) as  $m(k_r, z_r) = \|y_{\text{PR2}} - \sqrt{P_r} \mathbf{h}_{6k_r, z_r}\|^2$  then, the decision variable is  $D = -\|\mathbf{h}_{6k_r, z_r} - \mathbf{h}_{6\hat{k}_r^{\text{PR}}, \hat{z}_r^{\text{PR}}}\|^2 - 2\Re\{\mathbf{n}_{\text{PR2}}^H (\mathbf{h}_{6k_r, z_r} - \mathbf{h}_{6\hat{k}_r^{\text{PR}}, \hat{z}_r^{\text{PR}}})\}$  with  $m_D = -\|\mathbf{h}_{6k_r, z_r} - \mathbf{h}_{6\hat{k}_r^{\text{PR}}, \hat{z}_r^{\text{PR}}}\|^2$  and  $\sigma_D^2 = 2N_0 \|\mathbf{h}_{6k_r, z_r} - \mathbf{h}_{6\hat{k}_r^{\text{PR}}, \hat{z}_r^{\text{PR}}}\|^2$ . Therefore,  $\text{CPEP}^{\text{R} \rightarrow \text{PR}}$  can be given as

$$\text{CPEP}^{\text{R} \rightarrow \text{PR}} = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{\|\mathbf{h}_{6k_r, z_r} - \mathbf{h}_{6\hat{k}_r^{\text{PR}}, \hat{z}_r^{\text{PR}}}\|^2}{4N_0 \sin^2 \theta}\right) d\theta \quad (18)$$

APEP of the R  $\rightarrow$  PR link is calculated by taking the expected value of (18) over  $\mathbf{h}_{6k_r}$  as

$$\text{APEP}^{\text{R} \rightarrow \text{PR}} = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{\sin^2 \theta}{\sin^2 \theta + (c/4N_0)} \right)^{N_{\text{pr}}} d\theta \quad (19)$$

where  $c = d_3^{-v} \|z_r - \hat{z}_r^{\text{PR}}\|^2$  if  $k_r = \hat{k}_r^{\text{PR}}$ , else  $c = d_3^{-v} (\|z_r\|^2 + \|\hat{z}_r^{\text{PR}}\|^2)$  BEP of the R  $\rightarrow$  PR link is given by

$$P_b^{\text{R} \rightarrow \text{PR}} = \frac{1}{MN_t} \sum_{x_r} \sum_{\hat{x}_r} \text{APEP}^{\text{R} \rightarrow \text{PR}} \times \frac{e(x_r \rightarrow \hat{x}_r^{\text{PR}})}{\log_2(MN_t)} \quad (20)$$

where  $e(x_r \rightarrow \hat{x}_r^{\text{PR}})$  is the number of erroneous bits in detection of the PLNC mapped signal through the R  $\rightarrow$  PR link.  $P_b^{\text{ST} \rightarrow \text{PR}}$  can be derived with similar steps by a modification in the decision metric from (10).

Due to the network symmetry, BEP of SU is identical to that of PU for  $P_p = P_s$ . However, since R belongs to SU, for  $P_r = P_s$ , SU achieves the same BEP performance as PU with an additional power of 3 dB.

**Performance evaluation:** In this section, theoretical BEP results obtained in the previous section are compared with computer simulation results. It is assumed that cognitive cross network forms a rectangular area with the path-loss exponent  $\nu = 4$  and  $N_r = N_{\text{pr}} = 2$ . In all figures, straight, dashed and dotted lines represent theoretical results while markers stand for computer simulation results.

In Fig. 2, the BEP performance of PR is given for variable  $M$  and  $N_t$  by assuming  $d_1 = d_3 = 1$  and  $d_2 = 0.1$  and it is compared with that of the reference scheme employing  $M$ -PSK modulation instead of SM. When  $M = 2, N_t = 2$  for the proposed scheme, the reference scheme with  $M = 4, N_t = 1$  provides a slightly better BEP performance. However, it can be seen from Fig. 2 that the SM-based new scheme provides a considerably better BEP performance than the reference scheme for higher data rates.

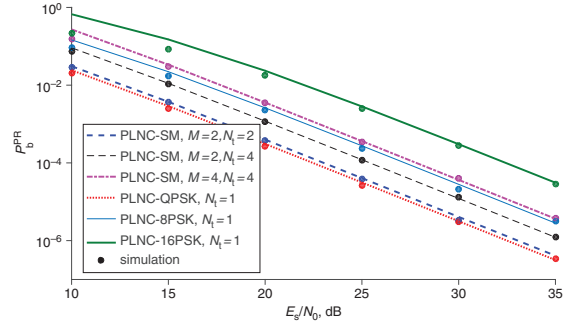


Fig. 2 BEP performance of PU for varying  $M$  and  $N_t$

In Fig. 3, the BEP performance of PR is given for  $d_2 = 0.5$  and the effect of the position of R is investigated for  $M = 4$  and  $N_t = 2$ . R is moved on the vertical direction in the rectangular area from the middle of transmitters to the middle of receivers by assuming a unity distance between the transmitters PT and ST. The best BEP performance at PR is obtained when R is at the centre of the rectangular area. Note also that in all cases the proposed scheme outperforms the scheme that performs direct transmission from PT to PR.

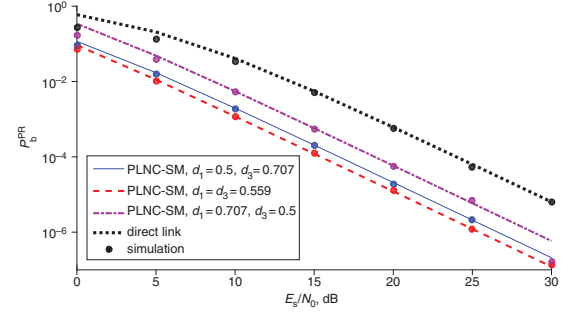


Fig. 3 Effect of relay position on BEP performance of PU

**Conclusion:** In this Letter, a cognitive cross network using SM at all nodes and PLNC at the relay, has been proposed. The superiority of the proposed scheme has been shown by comparisons with the classical  $M$ -PSK modulated as well as direct transmission schemes.

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One or more of the Figures in this Letter are available in colour online.

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