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Low Complexity Peak-to-Average Power Ratio Reduction in OFDM-IM

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***Abstract*—Orthogonal frequency division multiplexing with index modulation (OFDM-IM) employs the indices of the active subcarriers for information transmission, as an alternative to conventional OFDM. It offers high spectral efficiency and high energy efficiency in comparison to OFDM thanks to the information bits conveyed by IM. However, OFDM-IM has the drawback of high peak-to-average power ratio (PAPR) similar to OFDM, and this important problem has not been studied well in the literature. Active constellation extension (ACE), which is one of the well-known PAPR reduction methods, can be used to solve this drawback of OFDM-IM. Owing to the fact that this PAPR reduction method is less effective for OFDM-IM, we propose the extension of the constellation over inactive subcarriers through adding clipped signals over them. These subcarriers have a signal power limited by an upper bound, and this causes a slight degradation in the bit error rate (BER) performance. Computer simulation results demonstrate that our proposed method has a better PAPR reduction performance than the ACE method for OFDM and OFDM-IM while being more energy efficient with a very slight degradation in BER performance when a proper clipping threshold level is selected. Additionally, it is shown that the proposed method and ACE can be further combined, and this provides an improved PAPR reduction. In order to decrease the computational complexity of the PAPR reduction method to the linear-logarithmic level, smart gradient projection (SGP) is employed.**

***Index Terms*—Active constellation extension, OFDM-IM, peak-to-average power ratio, PAPR reduction, smart gradient projection.**

I. Introduction

Orthogonal frequency division multiplexing with index modulation (OFDM-IM) has been regarded as a candidate multi-carrier transmission scheme for spectrum- and energy-efficient next generation wireless communication systems [1]. In OFDM-IM, differently from conventional OFDM, both the indices of active subcarriers and data symbols at the active subcarriers convey information, and these subcarriers are activated according to the incoming bit sequences. OFDM-IM offers attractive advantages over conventional OFDM, such as better bit error rate (BER) performance at low-to-mid level spectral efficiency values, higher robustness to inter-carrier interference (ICI), better ergodic achievable rate, and being more flexible based on different channel conditions and system requirements [2].

Several methods have been proposed in the literature to solve the peak-to-average power ratio (PAPR) problem of OFDM [3]. Due to the nonlinearity of the power amplifier (PA), a high PAPR generates interference among the subcarriers, which causes a degradation in BER. The clipping and filtering (CF) method is one of the simplest PAPR reduction methods [4]. This method reduces PAPR significantly; however, it causes in-band and out-of-band (OOB) distortion that increase the overall BER. Tone reservation is a technique that reserves a small subset of tones, which are known by the transmitter and the receiver, to reduce the PAPR [5]. Consequently, this technique causes a decrease in data rate due to the fact that some tones are not used to convey information. Active constellation extension (ACE) is another efficient PAPR reduction technique in which the outer constellation points are wisely extended to decrease the PAPR [6]. This extension leads to an increase in the average transmitted power; however, the minimum Euclidean distance between constellation points remains the same.

Although OFDM-IM has appealing advantages over conventional OFDM, their PAPRs are almost the same by assuming input symbols with Gaussian distribution [7]. Therefore, OFDM-IM suffers from a high PAPR problem as OFDM. To solve this, the inactive subcarriers in OFDM-IM are used for PAPR reduction in [8]. The authors of [8] showed that this method outperforms selective mapping (SLM) and ACE methods. For effective operation of this technique, signal magnitudes of the inactive subcarriers have to be limited by an upper bound not to degrade the BER performance significantly. Finally, convex optimization is used to find the proper sample values for inactive subcarriers. However, the practical implementation of this method can be challenging, particularly for low-power devices, because of the required high computational cost for the PAPR reduction.

As an attractive candidate for conventional OFDM, efficient PAPR reduction techniques have to be introduced for OFDM-IM as well. However, this important problem has not been comprehensively explored in the literature. Although the existing PAPR reduction techniques for OFDM, such as the ACE method, can be also implemented for OFDM-IM, the inefficiency of these techniques for OFDM-IM due to its different symbol structure, constitutes the main motive of this study. In this paper, we propose the utilization of the inactive subcarriers within an extension region limited by an upper

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bound to decrease the PAPR differently from the scheme of [8]. However, using additional signals at the transmitter with the purpose of PAPR reduction produces interference among subcarriers. As pointed out in [9], this interference can be removed with suppressing alignment. On the other hand, we minimize the interference instead of removing it by utilizing the inactive subcarriers under a strict energy constraint not the degrade the BER performance. Additionally, it is shown that the proposed PAPR reduction method can be further combined with the ACE technique. Instead of convex optimization, a more practical iterative search algorithm is used to decrease the computational complexity of PAPR reduction. Our major contributions can be summarized as follows:

1) In order to effectively reduce the PAPR of OFDM-IM for low-power wireless communication systems, a technique with low computational complexity is proposed.
2) It has been shown that the proposed technique considerably outperforms ACE in PAPR reduction with a slight degradation (less than 0.5 dB for a target BER value) in BER performance.
3) To further reduce the PAPR, ACE and the proposed method are jointly implemented.

The rest of the paper is organized as follows. In Section II, the system model of the proposed model is introduced. In Section III, the ACE method explained briefly and the proposed method is described with illustrations. Then, computer simulation results are provided in Section IV. Finally, the paper is concluded in Section V. [1]

II. System Model

We consider the multicarrier transmission scheme given in Fig. 1. Here, a total of m information bits is divided into g parts each containing p bits, i.e., $m = pg$ and $g = N/n$, where N is the size of OFDM-IM block. In each subblock of length n, k out of n subcarriers are activated based on the first p_1 bits of the incoming p bits, where $p = p_1 + p_2$, $p_1 = \lfloor \log_2(C(n,k)) \rfloor$ and $p_2 = k\log_2(M)$. The remaining p_2 bits are reserved for M-ary constellation symbols. Consequently, the information bits are carried by the active subcarriers as well as their indices. In an OFDM-IM symbol, the set of active subcarriers is represented by

$$\mathcal{I} = \bigcup_{\beta=1}^{g} I_\beta, \tag{1}$$

$$I_\beta = \{i_{\beta,1}, \ldots, i_{\beta,k}\} \tag{2}$$

where $i_{\beta,l} \in \{1,2,...,n\}$ for $\beta = 1,2,...,g$ and $l = 1,2,...,k$. The size of $\mathcal{I}$ is K and $K = kg$ is the total number of

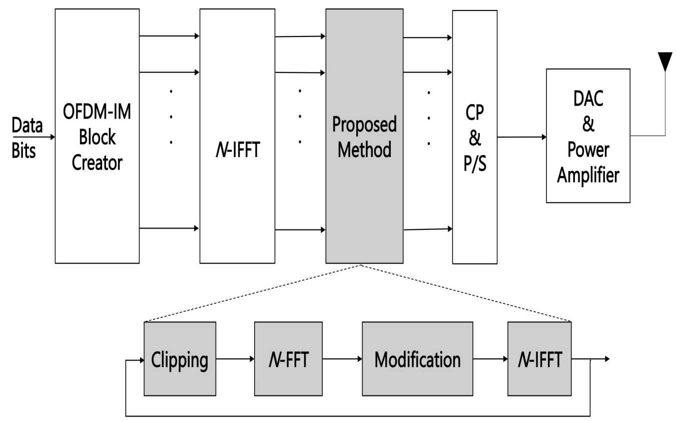

Fig. 1. Block diagram of the proposed method.

active subcarriers. The vector of M-ary data symbols in each subblock β is represented as

$$\mathbf{S}_\beta = [s_{\beta,1}\ s_{\beta,2}\ \ldots\ s_{\beta,k}]^T \tag{3}$$

where $s_{\beta,l} \in \mathbb{S}$ for $\beta = 1,2,...,g$, $l = 1,2,...,k$ and $E[|s_{\beta,l}|^2] = 1$. As a result, the OFDM-IM block is formed according to I_β and $\mathbf{S}_\beta$ as

$$\mathbf{X} = [X(1)\ X(2)\ \ldots\ X(N)]^T \tag{4}$$

where $X(j) \in \mathbb{S}$ for all $j \in \mathcal{I}$, otherwise $X(j) = 0$, $j = 1,2,...,N$. After the formation of the OFDM-IM block with dimensions $N \times 1$, the time domain (TD) OFDM-IM signal is obtained by taking the *inverse fast Fourier transform* (IFFT) of $\mathbf{X}$:

$$\mathbf{x} = \frac{N}{\sqrt{K}}\mathrm{IFFT}\{\mathbf{X}\} = \frac{1}{\sqrt{K}}\mathbf{F}_N^H\mathbf{X} \tag{5}$$

where $\mathbf{F}_N$ is the discrete Fourier transform (DFT) matrix with $\mathbf{F}_N^H\mathbf{F}_N = N\mathbf{I}_N$. The normalization factor of $(N/\sqrt{K})$ is used to ensure the same average transmission energy with conventional OFDM. For $\mathbf{x} = [x(1)\ x(2)\ \ldots\ x(N)]^T$, the PAPR is defined as

$$\mathrm{PAPR} = \frac{\max\limits_{j=1,\ldots,N} |x(j)|^2}{\frac{1}{N}\sum\limits_{j=1}^{N} |x(j)|^2}. \tag{6}$$

In the proposed method, the inactive subcarriers are employed to decrease the PAPR. In Section III, the proposed method, which further processes $\mathbf{x}$, will be explained in detail. After the formation of $\mathbf{x}$, a cyclic prefix (CP) of length C_p is included to the processed time-domain (TD) signal and parallel to serial (P/S) conversion is performed. After digital-and-analog conversion (DAC) and power amplification, the signal is transmitted over the selective Rayleigh fading channel.

At the receiver, the frequency-domain (FD) received signals are expressed by

[1] *Notation:* Bold, lowercase and capital letters denote the signal vectors in time domain and frequency domain, respectively. $C(n,k)$ denotes the binomial coefficient. $(.)^T$ and $(.)^H$ denote transposition and Hermitian transposition, respectively. $\|.\|_\infty$ denotes the infinity norm of a vector. The expected value and the floor function are denoted by $E[.]$ and $\lfloor . \rfloor$, respectively. $\mathbb{S}$ denotes the set of M-ary constellation symbols. $\mathbf{I}_N$ is the identity matrix with dimensions of $N \times N$. $\mathcal{CN}(0,\sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with variance σ^2.

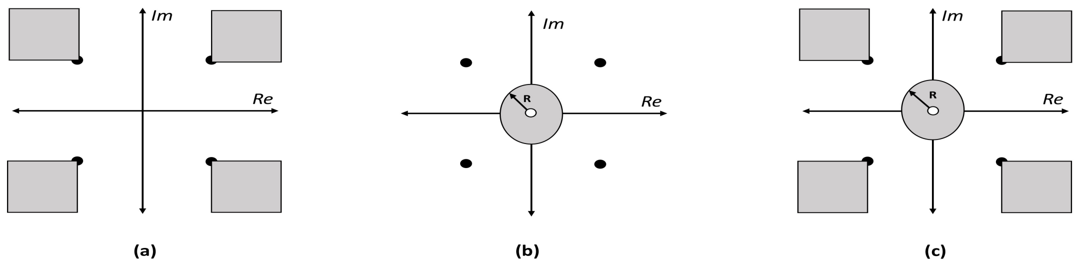

Fig. 2. Signal constellations for (a) the ACE method, (b) Method I and (c) Method II, for QPSK modulation. The markers of ○ and ● denote the constellation points used for the inactive and active subcarriers, respectively. The shaded parts show extension regions.

$$Y(j) = X(j)H(j) + W(j), \quad j = 1, 2, ..., N \tag{7}$$

where $W(j) \sim \mathcal{CN}(0, N_{0,F})$ is the additive white Gaussian noise sample and $H(j) \sim \mathcal{CN}(0, 1)$ is the channel fading coefficient for the jth subcarrier in the FD. The signal-to-noise ratio (SNR) is defined as $E_b/N_{0,T}$, where E_b is the average energy per bit, and $N_{0,T}$ is the noise variance in TD, given as $N_{0,T} = (N/K)N_{0,F}$. $\mathbf{H}$ is obtained by taking FFT of $\mathbf{h} = [h(1)\ h(2)\ \ldots\ h(v)\ 0\ \ldots\ 0]^T \in \mathbb{C}^{N\times 1}$, where v is the total number of channel taps. For the detection of active subcarrier indices and the corresponding data symbols, the maximum-likelihood detector [10] is implemented by

$$(\hat{I}_\beta, \hat{\mathbf{S}}_\beta) = \arg\min_{I_\beta, \mathbf{S}_\beta} \sum_{l=1}^{k} |Y_\beta(i_{\beta,l}) - H_\beta(i_{\beta,l}) s_{\beta,l}|^2 \tag{8}$$

where $Y_\beta(i_{\beta,l})$ and $H_\beta(i_{\beta,l})$ stand for the FD received signals and channel coefficients for the βth subblock, respectively.

III. PAPR Reduction In OFDM-IM

A. ACE Method

The ACE method uses the outer constellation points to decrease the PAPR. These points are extended without decreasing the minimum Euclidean distance between the constellation points, which are modified in a feasible region. With the extension of constellation points, the minimum distances between the constellation points remain the same, and a degradation in the BER performance is avoided. The PAPR reduction problem in the ACE method can be formulated as

$$\min_{\mathbf{c}} \|\mathbf{x} + \mathbf{c}\|_\infty^2 \equiv \min_{\mathbf{C}} \|\mathrm{IFFT}(\mathbf{X} + \mathbf{C})\|_\infty^2 \tag{9}$$

where $\mathbf{c} = [c(1)\ c(2)\ \ldots\ c(N)]^T$ is the added signal vector to reduce the PAPR and $\mathbf{C}$ is its FD representation. To find the signal vector of $\mathbf{c}$ that compensates the peak values in (9), a number of iterative methods, based on the clipping of the TD signal, exist for the ACE method, such as projection onto convex sets (POCS), approximate gradient-projection and smart gradient-projection (SGP) methods [6].

The ACE method can be used directly in OFDM-IM; however, it provides a low PAPR reduction since all of the subcarriers in an OFDM-IM symbol are not used for the purpose of IM. In other words, K subcarriers are utilized in OFDM-IM to reduce the PAPR while N subcarriers are used in OFDM, and this degrades the PAPR reduction performance of the ACE method for OFDM-IM. This is one of the main motivations of the proposed scheme.

B. Proposed Method

In order to reduce the PAPR effectively, we propose that the inactive subcarriers can take part in PAPR reduction by their extension, and this can be combined with the ACE method. The proposed two methods are illustrated for QPSK signal constellation in Fig. 2. In Method I, only the signals of inactive subcarriers are modified within the extension region limited by the radius of R, and Method II is the combination of Method I and the ACE method.

Here, the iterative process to decrease the PAPR can be formulated as

$$\mathbf{x}^{t+1} = \mathbf{x}^t + \alpha\mathbf{c} \tag{10}$$

where α is the convergence factor and t is the iteration index. To find the proper $\mathbf{c}$ and α, similar steps used with the ACE-SGP [6] are employed for the inactive subcarriers of OFDM-IM as follows:

1) First, deploy IFFT operation to the OFDM-IM block $\mathbf{X}$ to obtain $\mathbf{x}^t$ at the first step, i.e., for $t = 0$.
2) Clip any sample satisfying $|x(j)| > A$ to get the clipped signal $c(j)$ as

$$c(j) = \begin{cases} 0, & |x(j)| \leq A \\ Ae^{\angle\theta_j} - x(j), & |x(j)| > A \end{cases} \tag{11}$$

where A is the clipping threshold magnitude and θ_j is the phase angle of the jth sample. $c(j)$ is a signal that decreases the peak values to the clipping threshold level. Also, the clipping ratio (CR) can be expressed as

$$CR = 10\log_{10}\left(\frac{A^2}{E[|x(j)|^2]}\right). \tag{12}$$

3) Take the FFT of $\mathbf{c}$ to obtain $\mathbf{C}$.
4) **Method I:** Keep the samples of $\mathbf{C}$, corresponding to the inactive subcarriers in $\mathbf{X}$, unchanged, while nulling the remaining samples in $\mathbf{C}$, corresponding to the active subcarriers in $\mathbf{X}$.

Method II: Only control the active subcarriers and keep the samples of $\mathbf{C}$, which are in the feasible regions of ACE, unchanged, and set the others to zero. Then apply the Method I.

5) Take the FFT of the new $\mathbf{C}$ to obtain $\mathbf{c}$.
6) Find the convergence factor α with SGP as in [6].
7) Obtain $\mathbf{x}^{t+1}$ using the update formula $\mathbf{x}^{t+1} = \mathbf{x}^t + \alpha\mathbf{c}$.
8) Increase t by one. If t is equal to the predetermined maximum number of iterations L, finish the process. Otherwise, go back to Step 2.

As seen from Fig. 2, the signals carried by the inactive subcarriers are limited to take values from a circular region with radius of R. To guarantee that the magnitudes of the inactive subcarrier samples are lower than R in FD, a clipping is needed in FD with an IFFT/FFT operation. Instead of this clipping operation, which can cause new peak values that decrease PAPR reduction, choosing the proper CR can be sufficient to keep the samples as low as possible.

C. Computational Complexity

In the ACE method, SGP and POCS iterative search algorithms are used for the practical implementation. To decrease the total number of required iterations and to achieve a satisfactory PAPR reduction, the SGP approach provides a simpler and more effective solution. For this reason, a similar algorithm based on SGP is exploited. In our proposed method, only a few iterations are needed to perform an effective PAPR reduction. Each iteration requires an IFFT/FFT operation pair with the complexity of $\mathcal{O}(N\log_2 N)$. The determination of the convergence factor with SGP has a computational cost of $\mathcal{O}(N)$. Therefore, the computational complexity of the proposed method is in the order of $\mathcal{O}(2LN\log_2 N + LN)$, which is the same as that of the ACE-SGP method. A similar algorithm with the POCS approach can also be used for the proposed method; however, it requires more iterations. In comparison to [8], the proposed method decreases the computational complexity from $\mathcal{O}((N-K)N^2)$ to $\mathcal{O}(2LN\log_2 N + LN)$, which is a remarkable reduction for practical N values such as 512, 1024 and 2048. Since convex optimization [11] is employed for $N-K$ inactive subcarriers, the scheme of [8] has a considerably high computational complexity cost of $\mathcal{O}((N-K)N^2)$, where $N-K$ usually is chosen as $N/2$ to convey the maximum number of bits through IM.

IV. Performance Evaluation

In this section, we provide computer simulation results for the proposed method and make the comparison with ACE-SGP in conventional OFDM and OFDM-IM. For all comparisons, system parameters are taken as $N = 256$, $C_p = 32$ and $v = 10$. The concept of OFDM-IM is realized based on values of $k = 2$, $n = 4$ and $M = 4$. The results are obtained for randomly generated 10^5 OFDM symbols. Unless stated otherwise, CR is taken as 4.8 dB throughout the computer simulations.

Fig. 3 shows the PAPR reduction results of the ACE-SGP in OFDM and OFDM-IM, Method I and Method II. OFDM and OFDM-IM schemes have an identical PAPR as seen from

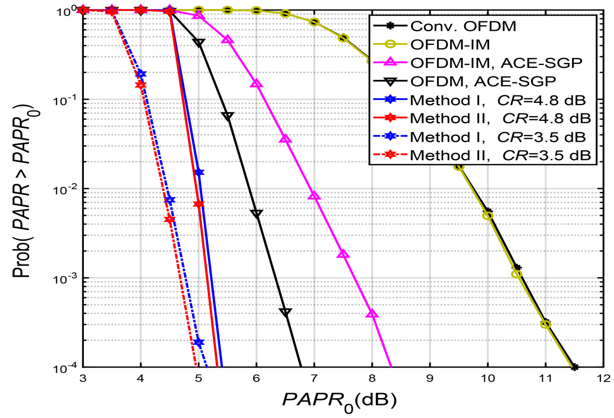

Fig. 3. PAPR reduction performance comparison of the proposed methods with $L = 2$ iterations for the different CR values.

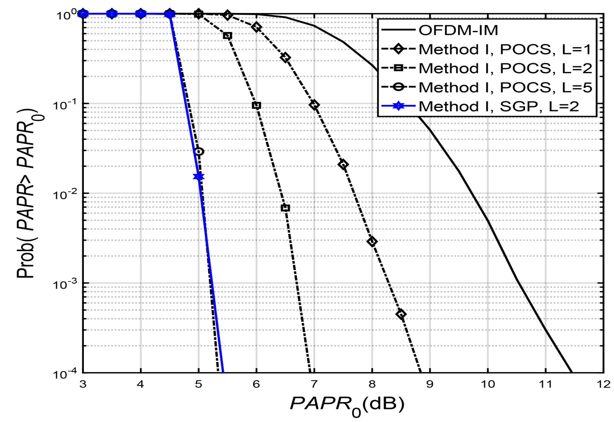

Fig. 4. PAPR reduction performance of the POCS approach for Method I .

Fig. 3. It is also shown that the ACE-SGP method achieves 4.7 dB PAPR reduction in OFDM, however, 3.3 dB in OFDM-IM, due to the inactive subcarriers of the latter. However, our proposed two methods for OFDM-IM provide an improved PAPR reduction compared to ACE-SGP aided OFDM and OFDM-IM. Finally, it is observed that Method II provides a slightly better PAPR than Method I due to additional ACE for active subcarriers. Because of the constellation extensions, the average power also increases and the increment values are 0.02 dB, 0.1 dB, 0.3 dB and 0.5 dB for Method I, Method II, OFDM with ACE-SGP, and OFDM-IM with ACE-SGP methods, respectively. Therefore, our proposed methods preserves more energy compared to the reference methods. Furthermore, the effects of different clipping ratios are also shown in Fig. 3. If CR decreases, a better PAPR reduction can be obtained; however, this increases the average power and the slope of the curve decreases. Consequently, the selection of the optimum CR has paramount importance for a better PAPR reduction performance and for different applications.

As mentioned previously, the POCS approach can be also used for the implementation of the proposed method. Fig.

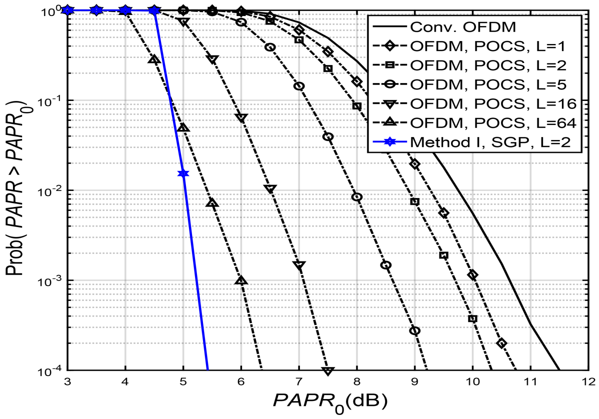

Fig. 5. PAPR reduction performance of the POCS approach for the ACE method.

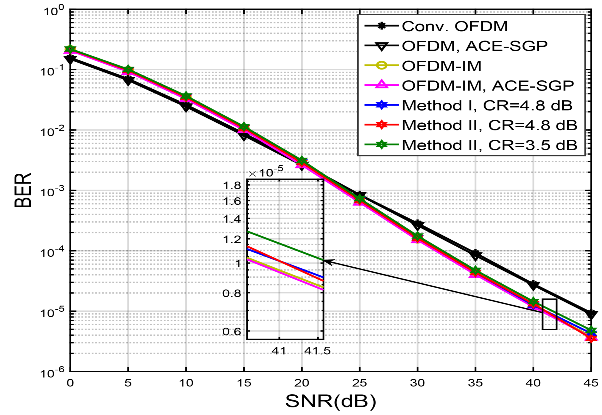

Fig. 6. The BER curves for the comparison of the proposed methods.

4 exhibits the PAPR curves of the proposed method with this approach. Here, the proposed method with two iterations provides the same PAPR reduction with the POCS approach with five iterations in the proposed method. Therefore, the SGP approach is employed to decrease the total number of iterations. From Fig. 4, we also observe that the POCS approach for our proposed methods converges faster than the POCS approach for the ACE method. Finally, Fig. 5 demonstrates that the POCS method in OFDM cannot reduce the PAPR even with 64 iterations to the level that Method I achieves with just two iterations for OFDM-IM.

As seen from Fig. 6, the ACE method provides a minor improvement in BER performance, since it increases the distance between constellation points without changing the minimum distance. In our proposed methods, the minimum distance is not guaranteed. Due to the fading channel and lower power signals, a slight BER degradation is observed from Fig. 6. More specifically, the proposed methods provide approximately 0.3 dB worse SNR compared to the plain OFDM-IM at a reference BER value of 10^{-5}. We also note that the BER increases when CR is decreased.

V. Conclusion

In this paper, we have proposed an efficient PAPR reduction technique for OFDM-IM with the overall computational complexity order of $\mathcal{O}(2LN\log_2 N + LN)$. It has been concluded that the inactive subcarriers can be extended within a limited region of radius R and this causes an almost negligible BER degradation. Furthermore, we have shown that this technique can be combined with the ACE method. Our proposed two methods have provided approximately 1.5 dB and 3 dB more PAPR reduction compared to the ACE method in OFDM and OFDM-IM, respectively. We have shown that Method II provides a better PAPR reduction performance, while Method I preserves more energy. The improvements provided by both methods have been verified compared to the ACE method.

In our future work, the proposed method can be implemented for higher order modulations. However, this will require a careful system design since R has to be smaller for higher order modulations not to degrade the BER performance in a significant manner, while the improvement in PAPR reduction should be adjusted with the proper selection of this parameter.

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