Quadrature Channel Modulation in the Presence of Channel Estimation Errors

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***Abstract*—This paper analyzes the effects of channel estimation errors on the error performance of the recently proposed quadrature channel modulation (QCM) over flat Rayleigh fading channels. A closed-form expression for the pairwise error probability (PEP) of the QCM scheme is derived in the presence of channel estimation errors, and used to calculate upper bound of the average bit error probability (ABEP) for M-QAM signalling. Computer simulation results are provided to corroborate the accuracy of the derived analysis with increasing signal-to-noise ratio (SNR). It has been shown by the computer simulation that the QCM scheme is considerably robust to channel estimation errors.**

***Index Terms*—Quadrature channel modulation, channel estimation errors, maximum likelihood detector, PEP/BEP analysis.**

I. Introduction

Index modulation (IM) techniques provide new ways to transmit information bits by using the indices and the on/off status of their transmission units such as transmit antennas, subcarriers, radio frequency (RF) mirrors, relays and so on. In other words, IM accomplishes the transmission of information bits by using new dimensions in addition to traditional modulations. Therefore, IM systems provide better spectral and energy efficiency with lower hardware complexity compared to traditional modulation techniques. Recently, several IM schemes and their performance have been studied in [1].

One of the basic type of the IM technique is known as spatial modulation (SM), which considers two-dimensional approach to transmit information bits by using the index of active antennas in addition to the traditional modulation techniques such as M-ary phase shift keying (M-PSK) or M-ary quadrature amplitude modulation (M-QAM) [2]. In this system, only one transmit antenna is activated by incoming random data bits at each symbol duration. Thus, SM not only increases the spectral efficiency logarithmically with the number of transmit antennas but also eliminates inter-carrier interference (ICI) and does not need transmit antenna synchronization compared to traditional MIMO solutions.

A modified version of SM known as quadrature spatial modulation (QSM) [3], allows one or two active transmit antennas simultaneously, which are activated by incoming information bits. For this scheme, the first $\log_2(M)$ bits of incoming data bits are used for ordinary M-QAM or M-PSK as in SM. The first active transmit antenna determined by $\log_2(n_T)$ bits of the incoming data bits for transmission of real part of the modulated signal. In the same way, the second active transmit antenna determined by another $\log_2(n_T)$ bits of the incoming data bits for transmission of imaginary part of the modulated signal. Thus, spectral efficiency of QSM increases logarithmically with the number of transmit antennas according to SM. However, the spectral efficiency of both SM and QSM increases logarithmically with the number of transmit antennas, in other words, the required transmit antennas increases exponentially. For this reason, a higher spectral efficiency can be only provided by a great number of transmit antennas for SM and QSM. Due to its aforementioned advantages, QSM has attracted the attention of the researchers, and it has been investigated in many recent studies from different perspectives, including performance analysis [4], [5], extension to cooperative networks [6], low-complexity detection [7] and enhanced designs [8].

An innovative approach to limit the number of transmit antennas has been recently proposed in [9] by the name of media-based modulation (MBM). The main idea of MBM is to create different channel fade realizations by using digitally controlled parasitic elements, which are known as radio frequency (RF) mirrors and mounted in a reconfigurable antenna. Thus, active channel state is determined by using the on/off status of the available RF mirrors according to n_{RF} bits of incoming random data bits, where n_{RF} is the number of RF mirrors. An advantage of MBM is that the spectral efficiency increases linearly with the number of used RF mirrors. This is in contrast with SM and QSM schemes where the spectral efficiency increases only logarithmically with the number of transmit antennas. Moreover, MBM can be easily combined with SM, QSM and other MIMO schemes.

Media-based modulation, when used together with QSM is referred to as the quadrature channel modulation (QCM), which has been recently proposed in [10]. In the QCM scheme, both indices of transmit antennas and status of RF mirrors are used for transmission of information bits, while the only status of RF mirrors and the index of transmit antennas are used in MBM and QSM, respectively. Therefore, the spectral efficiency of QCM is higher than MBM and QSM. In [10], three types of QCM schemes are provided. In the first QCM

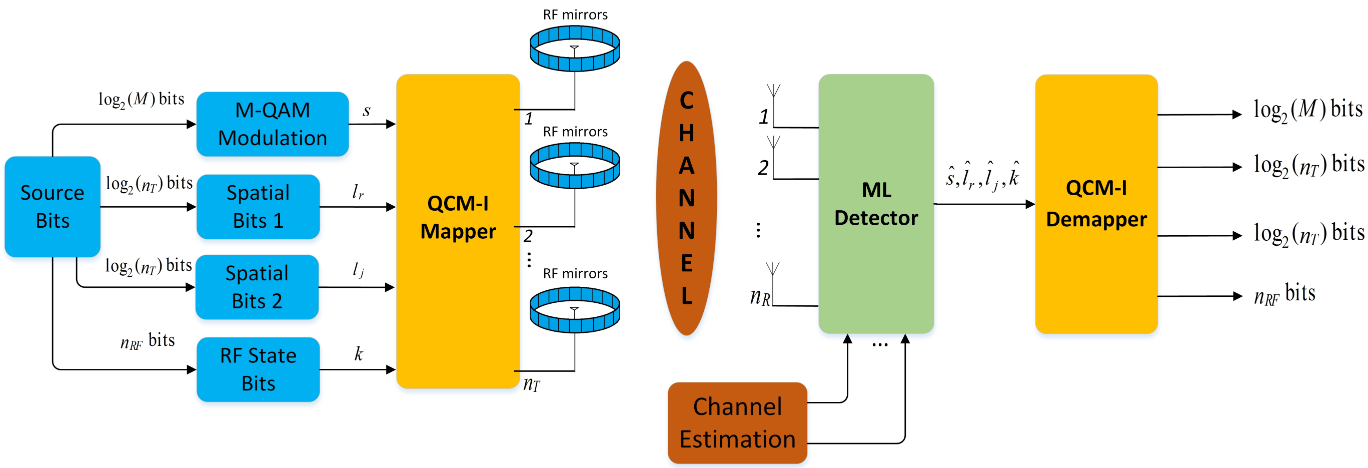

Fig. 1. Block diagram of the QCM-I scheme for an $n_T \times n_R$ MIMO system.

scheme, which is called QCM-I, the first $\log_2(M)$ bits of incoming data bits are used to determine ordinary M-QAM symbol and the next $2\log_2(n_T)$ bits are used to determine the transmit antennas for transmission of real and imaginary part of M-QAM symbol as in QSM. In addition to QSM, the last n_{RF} bits of incoming data bits are used to determine the on/off status of RF mirrors as in MBM, in which RF mirrors are mounted around each transmit antenna unit. In other words, active channels for transmission of real and imaginary part of M-QAM symbol are determined by both active antenna and status of RF mirrors. Thus, incoming data bits are mapped as shown in Fig. 1.

In this paper, we investigate the negative effects of channel estimation errors on the performance of QCM, which has not been reported analytically before. In [11] and [12], the performance of SM and MBM have been studied in the presence of imperfect CSI. We study the least squares (LS) estimation based on maximum-likelihood (ML) detection for the QCM-I scheme under the channel estimation errors. Also, we calculate a closed form expression for the pairwise error probability (PEP) of the QCM-I scheme and it is used to derive the upper bound of the average bit error probability (ABEP) for ordinary M-QAM scheme. The remainder of this paper is organized as follows. System model is given in Section II. Our theoretical approach to the ABEP calculation of QCM-I is presented in Section III. Numerical examples are provided in Section IV. Finally, the paper is concluded in Section V.

II. System Model

The system model of the QCM-I scheme is shown in Fig. 1. We consider the QCM-I scheme with n_T transmit and n_R receive antennas that operates over a Rayleigh flat fading channel using M-QAM. Furthermore, n_{RF} RF mirrors are mounted around each transmit antenna in order to create different channel realizations according to the incoming information bits. The channel fading coefficient between the tth transmit antenna corresponding to the status of RF mirrors (kth state) and the rth receive antenna is denoted by $h_r^{t,k}$, which is distributed as $\mathcal{CN}(0,1)$.

Assume a total of

$$\eta = 2log_2(n_T) + log_2(M) + n_{RF} \tag{1}$$

bits enter the QCM-I mapper at each transmission interval. The mapper groups incoming information bits as shown in Fig. 1. For example, for $M=16$, $n_T=4$, $n_R=4$ and $n_{RF}=2$, ten ($\eta=10$) information bits are transmitted by the transmitter in each transmission interval. First four bits determine the 16-QAM symbol (x) and the next four bits determine the index of the first and the second transmit antennas (l_R and l_J) for the transmission of real and imaginary parts of this 16-QAM symbol, while the last two bits determine channel state (k), which corresponds to the following on/off status ($0 \rightarrow$ off and $1 \rightarrow$ on) of two RF mirrors. Thus, the modulated symbol is denoted by $q = (l_R, l_J, k, x)$ where x is transmitted over the l_Rth and l_Jth transmit antennas with the kth channel state.

The signal model of QCM-I scheme is given by [10]

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{2}$$

where $\mathbf{y} \in \mathbb{C}^{n_R \times 1}$, $\mathbf{H}$, $\mathbf{s}$ and $\mathbf{n} \in \mathbb{C}^{n_R \times 1}$ respectively denote received signal vector, channel matrix, transmission vector and additive white Gaussian noise (AWGN) vector, whose entries are distributed by $\mathcal{CN}(0, N_0)$. $\mathbf{H}$ and $\mathbf{s}$ are defined as follows

$$\mathbf{H} = \begin{bmatrix}\mathbf{h}^{1,1} & \mathbf{h}^{1,2} & \ldots & \mathbf{h}^{1,k} & \mathbf{h}^{2,1} & \ldots & \mathbf{h}^{t,k} & \ldots & \mathbf{h}^{n_T 2^{n_{RF}}}\end{bmatrix},$$

$$\mathbf{h}^{t,k} = \begin{bmatrix} h_1^{t,k} & h_2^{t,k} & \ldots & h_{n_R}^{t,k}\end{bmatrix}^T \tag{3}$$

where $t \in \{1, \ldots, n_T\}$, $k \in \{1, \ldots, 2^{n_{RF}}\}$ and

$$\mathbf{s} = [\ldots \ \underbrace{|0 \ \ldots \ \overbrace{s_r}^{l_r} \ \ldots \ 0|}_{\text{Ch. state } k_r} \ \ldots \ \underbrace{|0 \ \ldots \ \overbrace{js_j}^{l_j} \ \ldots \ 0|}_{\text{Ch. state } k_j} \ \ldots]^T \tag{4}$$

where $k_r = k_j$ for the QCM-I scheme, l_r and l_jth entries of $\mathbf{s}$ are non-zero only and $(.)^T$ stands for the transposition of a vector. Dimensions of $\mathbf{H}$ and $\mathbf{s}$ are $N_r \times 2^{n_{RF}} n_T$ and $2^{n_{RF}} n_T \times 1$, respectively. We assume that $h_r^{t,k}$ is the component of $\mathbf{H}$.

Assuming the QCM-I symbol $\mathbf{s}$ is transmitted and it is eroneously detected as $\hat{\mathbf{s}}$ at the receiver, while channel state information is perfectly known, the conditional PEP is given by [13]

$$P(\mathbf{s} \to \hat{\mathbf{s}}|\mathbf{H}) = Q\left(\sqrt{\frac{\|\mathbf{H}(\mathbf{s}-\hat{\mathbf{s}})\|^2}{2N_0}}\right). \tag{5}$$

In practice, fading coefficients have to be estimated at the receiver for the detection of the transmitted signal vector. Fading coefficient of channel is obtained by using channel estimation techniques. Pilot-based (pilot symbols should be known at the receiver a priori) approaches are widely used to estimate the fading coefficient. If the channel is estimated with least squares (LS), the estimated fading coefficient has the form

$$\hat{h}_r^{t,k} = h_r^{t,k} + \epsilon_r^{t,k} \tag{6}$$

where $\epsilon_r^{t,k}$ denotes the channel estimation error, which is independent of $h_r^{t,k}$ and distrubuted according to $\mathcal{CN}(0, {\sigma_e}^2)$ [14]. Thus $\hat{h}_r^{t,k}$ is dependent on $h_r^{t,k}$ with the correlation coefficient $\rho = 1/\sqrt{1+{\sigma_e}^2}$ and distribution of $\hat{h}_r^{t,k}$ becomes $\mathcal{CN}(0, 1+{\sigma_e}^2)$. We assume that ρ is known by the receiver. Consequently, $\hat{\mathbf{H}}$ and $\mathbf{E}$ denote the estimated channel and the estimation error matrix, respectively. From (6), $\hat{\mathbf{H}}$ can be written as

$$\hat{\mathbf{H}} = \mathbf{H} + \mathbf{E}. \tag{7}$$

When QCM-I symbol $q = (l_R, l_J, k, x)$ is transmitted, the mean and variance of the received signal $y_r, r = 1, \dots, n_R$ conditioned on $\hat{h}_r^{t,k}$ under the channel estimation error are given as [11]

$$\begin{aligned} E\{y_r|\hat{h}_r^{t,k}\} &= \rho^2 \hat{h}_r^{t,k} x \\ Var\{y_r|\hat{h}_r^{t,k}\} &= N_0 + (1-\rho^2)|x|^2. \end{aligned} \tag{8}$$

At the receiver side of these systems, ML detector is performed by the following minimization

$$\begin{aligned} [\hat{l}_R, \hat{l}_J, \hat{k}, \hat{x}] = \arg \min_{l_R, l_J, k, x} & \frac{\|\mathbf{y}-\hat{\mathbf{H}}\mathbf{s}\|^2}{N_0 + (1-\rho^2)|x|^2} \\ & + \; n_R \log(N_0 + (1-\rho^2)|x|^2). \end{aligned} \tag{9}$$

III. Bit Error Probability Analysis

In this section, we derive a closed form expression for the pairwise error probability of the QCM-I scheme in the presence of channel estimation errors, and it is used to calculate the upper bound of ABEP for M-QAM signaling.

The conditional PEP of $\mathbf{s}$ being decoded as $\hat{\mathbf{s}}$ can be written as

$$P(\mathbf{s} \to \hat{\mathbf{s}}|\hat{\mathbf{H}}) = P(\|\mathbf{y}-\hat{\mathbf{H}}\hat{\mathbf{s}}\|^2 < \|\mathbf{y}-\hat{\mathbf{H}}\mathbf{s}\|^2) \tag{10}$$

and after simple manipulation, we obtain

$$P(\mathbf{s} \to \hat{\mathbf{s}}|\hat{\mathbf{H}}) = P(|\hat{\mathbf{H}}(\mathbf{s}-\hat{\mathbf{s}})|^2 - 2\Re\{y^* \hat{\mathbf{H}}(\mathbf{s}-\hat{\mathbf{s}})\}) > 0. \tag{11}$$

Thus, the conditional PEP (CPEP) of QCM-I can be written as

$$P(\mathbf{s} \to \hat{\mathbf{s}}|\hat{\mathbf{H}}) = Q\left(\sqrt{\frac{\|\hat{\mathbf{H}}(\mathbf{s}-\hat{\mathbf{s}})\|^2}{2(N_0 + (1-\rho^2)|x|^2}}\right). \tag{12}$$

Using the alternative form of the Gaussian Q-function [15], (12) can be written as

$$P(\mathbf{s} \to \hat{\mathbf{s}}|\hat{\mathbf{H}}) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(\frac{\|\hat{\mathbf{H}}(\mathbf{s}-\hat{\mathbf{s}})\|^2}{4\sin^2\theta(N_0 + (1-\rho^2)|x|^2}\right) \mathrm{d}\theta. \tag{13}$$

We define $d_r \triangleq \|\hat{\mathbf{H}}(\mathbf{s}-\hat{\mathbf{s}})\|^2$ and moment generating function (MGF) [15] of d_r, given by

$$M_{d_r}(t) = \frac{1}{1 - \|\mathbf{s}-\hat{\mathbf{s}}\|^2(1+{\sigma_e}^2)t}. \tag{14}$$

Finally, unconditional PEP (UPEP) is obtained as

$$P(\mathbf{s} \to \hat{\mathbf{s}}) = \frac{1}{\pi}\int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta + \dfrac{\|\mathbf{s}-\hat{\mathbf{s}}\|^2}{4(N_0 + (1-\rho^2)|x|^2}}\right)^{n_R} \mathrm{d}\theta \tag{15}$$

which has a closed form solution in [15] as follows

$$P(\mathbf{s} \to \hat{\mathbf{s}}) = \left(\frac{1-\mu(c)}{2}\right)^{n_R} \sum_{k=1}^{n_R-1} \binom{n_R-1+k}{k} \left(\frac{1+\mu(c)}{2}\right)^k \tag{16}$$

where $c = \dfrac{\|\mathbf{s}-\hat{\mathbf{s}}\|^2}{4(N_0 + (1-\rho^2)|x|^2}$ and $\mu(c) = \sqrt{\dfrac{c}{1+c}}$.

After the calculation of the closed form UPEP of the QCM-I scheme under channel estimation errors, an upper bound on the ABEP based on union bounding principle can be obtained as

$$P_b \le \frac{1}{2^\eta} \sum_{i=1}^{\eta} \sum_{\substack{j=1 \\ j \ne i}}^{\eta} \frac{p(\mathbf{s}_i \to \mathbf{s}_j) e(\mathbf{s}_i, \mathbf{s}_j)}{\eta} \tag{17}$$

where $e(\mathbf{s}_i, \mathbf{s}_j)$ is the number of bit errors corresponding pairwise error event.

IV. Simulation Results and Discussion

In the following, the performance of the QCM-I technique, under channel estimation errors, is evaluated analytically as well as with computer simulations and compared with the case of perfect-channel state information (P-CSI).

The ABEP is depicted as a function of SNR in the computer simulations. The SNR is defined as SNR $= E_b/N_0$, where E_b is the average transmitted energy per bit. For simulations, at least 10^8 bits have been transmitted for each considered SNR value. We assumed $n_R = 4$ for all simulations. Furthermore, we consider Gray mapping for M-QAM symbols and natural mapping for indices of channel states and transmit antennas.

Computer simulation results are shown in Fig. 2, for the QCM-I scheme with $n_T = 4$, $n_{RF} = 2$ and 16-QAM at 10 bits/s/Hz for fixed σ_e^2 values (0.003, 0.005 and 0.007).

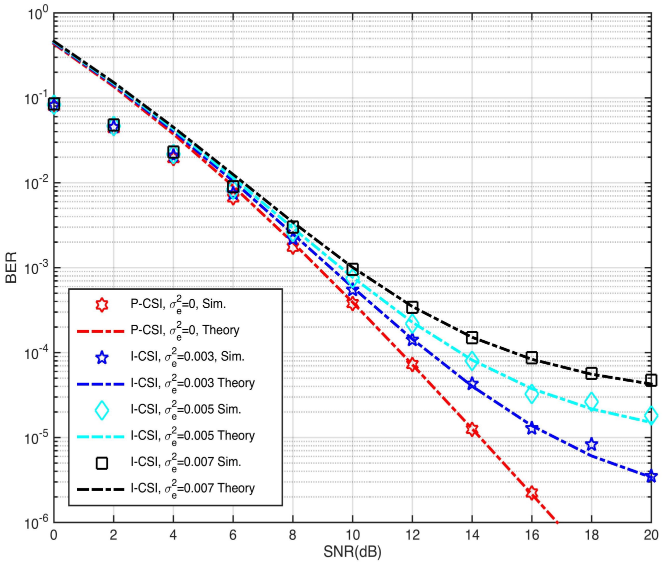

Fig. 2. BER performance of QCM-I with $n_T = 4$, $n_{RF} = 2$, 16-QAM, $\eta = 10$ bpcu and ML detector with fixed $\sigma_e^2 = \{0, 0.003, 0.005, 0.007\}$.

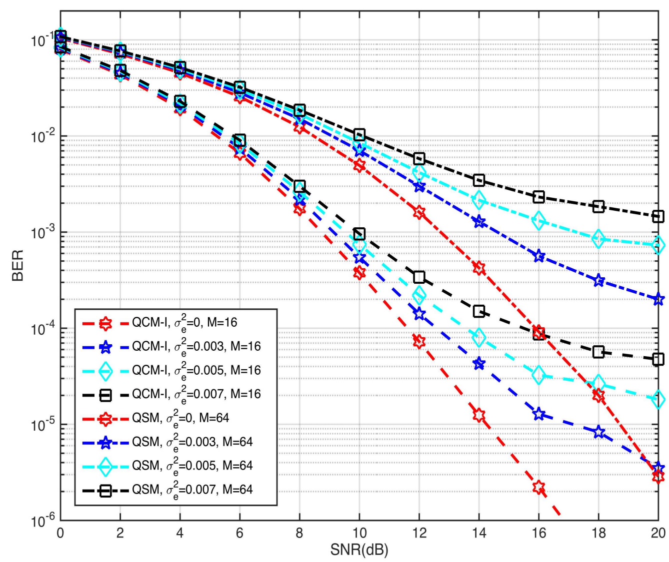

Fig. 3. BER performance of QCM-I and QSM with $n_R = 4$, $\sigma_e^2 = \{0, 0.003, 0.005, 0.007\}$, $\eta = 10$ bpcu and ML detector. QCM: $n_{RF} = 2$, $n_T = 4$, 16-QAM; QSM: $n_T = 4$, 64-QAM.

As seen from Fig. 2, the theoretical upper bounds provided by (17) become extremely tight with high SNR for all σ_e^2 values. Also, it is observed that the BER performance is affected negatively under channel estimation errors, and this negative effect increases with the variance of the estimation error, σ_e^2. As an example, at a BER of 10^{-4}, the performance degradation with imperfect CSI (I-CSI) (i.e, $\sigma_e^2 \neq 0$) compared to P-CSI (i.e, $\sigma_e^2 = 0$) is about 0.5 dB, 1 dB and 2 dB for σ_e^2 values of 0.003, 0.005 and 0.007, respectively. Consequently, we observe that the QCM-I scheme is quite robust to the channel estimation errors.

In Fig. 3, we show the performance of the QCM-I and QSM schemes under the channel estimation errors. Both schemes use $n_R = 4$ and $\sigma_e^2 = \{0, 0.003, 0.005, 0.007\}$, $\eta = 10$ bpcu, and ML detector. The schemes configured as follows: $i)$ QCM-I: $n_{RF} = 2$, $n_T = 4$, 16-QAM; $ii)$ QSM: $n_T = 4$, 64-QAM. It is observed that the BER performance of QCM-I scheme is better than QSM scheme for same number of transmit antennas and bpcu. As an example, to achieve 10^{-3} BER at $\sigma_e^2 = 0.007$, the required SNR is 10 dB and more than 20 dB for QCM-I and QSM, respectively.

Similarly, in Fig. 4, we compare the performance of QCM-I and MBM schemes under the channel estimation errors. Both schemes use $n_R = 4$, $\sigma_e^2 = \{0, 0.003, 0.005, 0.007\}$, $\eta = 10$ bpcu, and ML detector. The schemes configured as follows: QCM-I: $n_{RF} = 2$, $n_T = 4$, 16-QAM; MBM-I: $n_T = 1$, $n_{RF} = 2$, 256-QAM; MBM-II: $n_T = 1$, $n_{RF} = 10$. Similar behaviors as in Fig. 3 can be observed in Fig. 4. For example, to achieve 10^{-2} BER at $\sigma_e^2 = 0.007$, the required SNR is 6 dB and 16 dB for QCM-I and MBM-I, respectively. On the other hand, MBM-II provides significant BER performance compared to QCM-I scheme and MBM-I.

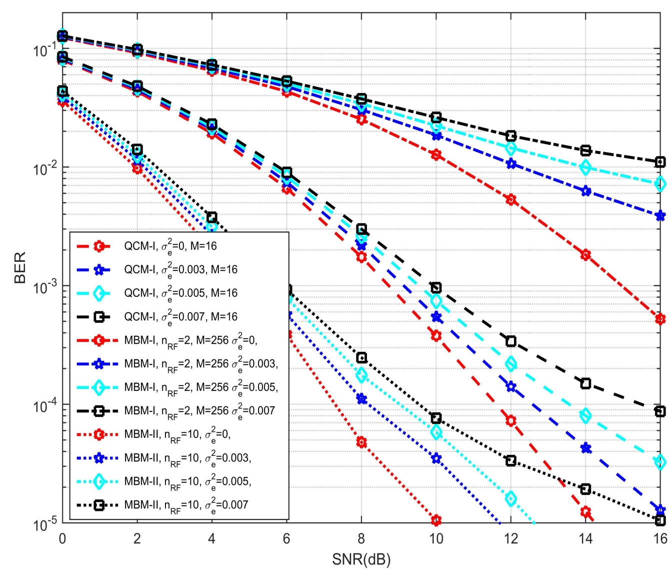

Fig. 4. BER performance of QCM-I and MBM with $n_R = 4$, $\sigma_e^2 = \{0, 0.003, 0.005, 0.007\}$, $\eta = 10$ bpcu and ML detector. QCM: $n_{RF} = 2$, $n_T = 4$, 16-QAM; MBM: $n_T = 1$, $i)$ $n_{RF} = 2$, 64-QAM, $ii)$ $n_{RF} = 10$.

This noticable performance is due to high data rates, while the traditional modulation order is increasing, the distance between their constellation points decreases. However, MBM-II, without traditional modulation, can increase the data rate while keeping the same distance between the transmission vectors, at the expense of ten RF mirrors.

V. Conclusion

QCM is a novel MIMO scheme reported recently. The performance of the QCM-I system over flat Rayleigh MIMO channels with imperfect CSI has been analyzed and studied for M-QAM signal constellations in this paper. Through computer simulations, we have shown that the derived PEP expression was accurate and the average BEP upper bounds were very tight with increasing SNR. Also, it has been shown via computer simulations as well as theoretical ABEP calculations that the QCM-I scheme is pretty robust to the channel estimation errors. Our future work may focus on correlated channel realizations and different channel estimation methods for the QCM system.

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