Cooperative Space Shift Keying Media-Based Modulation With Hybrid Relaying

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Abstract—In this paper, we propose two novel cooperative space shift keying media-based modulation (SSK-MBM) schemes. In the first scheme, a multiantenna source communicates with multiple relays and the destination by applying SSK-MBM, i.e., by altering the activated antenna and the ON/OFF status of radio-frequency mirrors. Moreover, multiantenna relays have the capability of hybrid relaying, which means that the relays that correctly decode the source’s signal take part in the transmission by applying the SSK-MBM technique; however, the relays that erroneously decode the source’s signal amplify and forward their received signal to the destination. In the second scheme, we consider a novel cooperative SSK-MBM scheme with a hybrid relay selection in which a multiantenna source communicates with a selected relay and the destination by applying SSK-MBM. Error probability expressions for the proposed schemes are derived. Comprehensive computer simulations show that the derived analytical expressions are in agreement with the numerical results and show that the performance of hybrid relaying scheme is superior to that of the decode-and-forward as well as the amplify-and-forward techniques. It is shown that the proposed SSK-MBM systems outperform the conventional cooperative single-input multiple-output and spatial modulation systems for particularly high modulation orders and sufficient number of receive antennas at the destination.

Index Terms—Cooperative relays, hybrid relaying (HR), hybrid relay selection (HRS), media-based modulation (MBM), space shift keying (SSK).

I. INTRODUCTION

N ext-generation wireless networks are expected to not only ensure uninterrupted and reliable communications, but also offer very high level of spectral and energy efficiency. In this regard, index modulation (IM) techniques have attracted remarkable attention due to their attractive advantages including better error performance and improved energy/spectral efficiency over conventional modulation schemes. The basic idea of IM is based on carrying additional information by the indices of the building blocks of the target communication systems [1]. Such building blocks can be either antennas in a multiantenna communication system or subcarriers in an orthogonal frequency-division multiplexing (OFDM) scheme. Spatial modulation (SM) and space shift keying (SSK) are two well-known IM techniques, which exploit the indices of transmit antennas to convey information bits [2], [3]. OFDM-IM is another well-known practice of IM techniques, which uses the indices of OFDM subcarriers to carry additional information bits [4]. In SM and SSK, depending on the one-to-one mapping between information bits and antenna indices, the transmitter activates typically only one transmit antenna and the other antennas remain silent. Therefore, interchannel interference is completely avoided, interantenna synchronization requirement among the transmit antennas is eliminated, and transceiver complexity is reduced. Moreover, the studies on the practical implementation of SM/SSK techniques show that these techniques are practically capable of exploiting the aforementioned advantages as well as outperforming conventional schemes, such as spatial multiplexing in terms of error performance [5]. Unlike SM, SSK does not carry information using conventional digital modulation parameters, such as amplitude, frequency, and phase, as well as further decreases the transceiver complexity since it can be implemented with a very simple hardware that does not require even a radio-frequency (RF) chain [6].

On the other hand, recently proposed media-based modulation (MBM) is a novel IM technique, which exploits the ON/OFF status of RF mirrors as an additional source of information [7], [8]. The RF mirrors contain PIN diodes and these mirrors can be turned ON or OFF according to the information bits to alter the far-field radiation pattern of a reconfigurable antenna. In MBM, switching between the ON/OFF status of RF mirrors enables unique mirror activation patterns (MAPs) as well as channel fade realizations, and the indices of unique MAPs are used to convey information. Similar to other IM techniques, better error performance and improved energy/spectral efficiency are obtained by MBM over conventional modulation schemes and it further outperforms SM in terms of error performance for high digital modulation orders without sacrificing the advantages provided by IM techniques [8]. The spectral efficiency of MBM systems increases linearly with the increasing number of RF mirrors, unlike SM and SSK, in which the spectral efficiency is limited by the logarithm of the number of...
transmit antennas. These outstanding advantages of MBM have attracted the attention of many researchers [9]–[11]. Moreover, the promising advantages of SM, SSK, and MBM techniques have inspired many researchers to combine them under various designs [8]–[12].

Cooperative relaying leverages the transmission reliability as well as provides broader coverage for wireless communication networks by introducing additional diversity through independent relaying links and enabling the decomposition of long links between the source and the destination into shorter links [13]. Hence, it improves the error performance and efficiency of the battery life of communicating devices as well as mitigates the effect of fading on the network. Two well-known forwarding methods of cooperative relaying are amplify-and-forward (AF) and decode-and-forward (DF), in which the error performance is limited by noise amplification and decoding failure at the relay, respectively. In order to cope with such limitations, many researchers have come up with new solutions and proposed novel cooperative schemes [14]–[16]. A more detailed literature review on such schemes can be found in [17] for the interested readers. One of the efficient solutions, which has attracted the researchers’ attention considerably, to overcome the limitations of the well-known forwarding methods of cooperative relaying is hybrid relaying (HR) [18]. In HR, the relays are capable of employing both the AF and DF methods according to their decoding result. For example, if a relay decodes its received signals without failure, then it employs the DF method whereas the AF method is employed if a failure occurs during the relay’s decoding procedure. On the other hand, the activation of all available relays by using orthogonal subchannels reduces the data rate and limits the throughput of cooperative networks. In this context, relay selection, which provides a better error performance compared to the single-relay transmission and avoids the degradation in the spectral efficiency compared to the multi-relay transmission, can further improve the efficiency of these networks [19]. In this regard, hybrid relay selection (HRS) can be considered as a promising technique to improve the efficiency of HR networks [17].

To exploit the advantages of both IM and cooperative networks, researchers have proposed various schemes [20]–[25]. In [20], Mesleh et al. investigate the error performance of a cooperative SSK scheme with AF and DF relays. SSK-aided AF and DF relaying with relay selection has been studied in [21] and [22], respectively. In [23], transmit antenna selection technique [26] is applied to cooperative SSK with multiple DF relays and the error performance of this system is investigated. Mesleh and Ikki [24] propose a cooperative SM system with multiple DF relays. A distributed SM protocol, in which the index of the relays conveys information, is proposed in [25]. On the other hand, to the best of author’s knowledge, MBM has not been considered in a cooperative scheme yet and HR and HRS techniques have not been studied as well in a cooperative IM scheme.

Motivated by all that has been discussed so far, in this paper, we propose two novel SSK-MBM schemes with HR and HRS in order to exploit the benefits of IM techniques as well as to obtain cooperative diversity gain through independent relaying links. Moreover, our proposed schemes exploit both the benefits of AF and DF relaying combined with the promising potential of SSK and MBM techniques in a novel fashion since, to the best of authors’ knowledge, the error performance of MBM in cooperative networks has not been reported and the performance of SSK with HR and HRS has not been investigated yet. Our novel contributions can be summarized as follows.

1) A cooperative SSK-MBM scheme with HR is proposed. In this scheme, the source and the relays are equipped with multiple transmit antennas and RF mirrors, while the destination is equipped with multiple receive antennas. The relays are capable of employing both AF and DF methods and forward their received signals according to their decoding results.

2) We propose a cooperative SSK-MBM scheme with HRS in order to increase the data rate and to reduce the complexity of the HR scheme. In this scheme, the source and the relays are equipped with multiple transmit antennas and RF mirrors, while the destination is equipped with multiple receive antennas and the selected relay follows the HR principle as in the previous scheme. Here, unlike SSK-MBM with HR, a relay, which can operate at either AF or DF mode, is selected with respect to pairwise error probability (PEP) of the system.

3) We derive exact PEP as well as approximate average bit error probability (ABEP) expressions and conduct asymptotic error probability analyses for the proposed systems. The derived expressions are shown to become consistent with computer simulation results.

4) Our numerical results indicate that the proposed SSK-MBM systems outperform SSK-MBM with conventional (AF or DF) relaying methods. It is also demonstrated that the proposed schemes outperform conventional digital modulation schemes and SM schemes with HR.

5) The proposed schemes have a general multiple-input and multiple-output structure with multiple transmit and receive antennas, which contains the MBM and SSK schemes as its special cases. They also apply an HR method, which can also be considered as the generalization of AF and DF methods.

II. SSK-MBM WITH HR

A. System Model

We consider an SSK-MBM scheme with cooperative relays as illustrated in Fig. 1. In this scheme, the source (S) is equipped with a single RF chain, $N_t$ transmit antennas, and $N_{rf}$ RF
mirrors enabling data transmission through available antenna indices and MAPs. Such patterns can be obtained by changing the index of the activated transmit antennas and the ON/OFF status of the RF mirrors according to the incoming data bits at the transmitter. Moreover, each relay $R_i$, $i = 1, \ldots, M$, has the same transmission capabilities as $S$ with a single transmit RF chain, $N_t$ transmit antennas, and $N_f$ RF mirrors; however, only one of the antennas can be used for signal reception at $R_i$, since we assume that it has only one receive RF chain. On the other hand, the destination (D) is a more sophisticated device with $N_d$ receive antennas and RF chains. It is assumed that a direct link transmission between S and D exists and perfect channel state information regarding the S–Ri and S–D links is available at D.

The transmission occurs in a two-stage protocol. At the first stage, S transmits its SSK-MBM signal to the relays and D. Suppose that $l, l' \in \{1, \ldots, N_s\}$, and $k, k' \in \{1, \ldots, 2N_f\}$, denote the activated transmit antenna index and the index of the MAP corresponding to the ON/OFF status of RF mirrors, the received signal at $R_i$ and the vector of received signals at D from S can be written, respectively, as

$$y_{SR}^i = \sqrt{E_S} h_{l,k}^i + n_{SR}^i$$

(1)

$$y_{SD}^i = \sqrt{E_S} h_{l,k}^i + n_{SD}^i$$

(2)

where $E_S$ is the average energy of the transmitted signal from S and $h_{l,k}^i$ is the channel fading coefficient between S and $R_i$ corresponding to the $l$th transmit antenna and $k$th MAP. More explicitly, considering that we have $N_t$ transmit antennas, each of them having $F = 2N_f$ available MAPs, $h_{l,k}^i$ is the $(F(l - 1) + k)$th element of the $S$–$R_i$ channel vector $h_{SR}^i \in \mathbb{C}^{1 \times N_s F}$, which can be expressed as in (3), shown at the bottom of this page.

Therefore, according to incoming data bits, one of the channel fading coefficients in $h_{SR}^i$ is received by $R_i$ and a total of $p = \log_2 N_s + N_f$ bits can be transmitted from S during a transmission interval. Similarly, $h_{l,k}^i \in \mathbb{C}^{N_d \times 1}$ denotes the $(F(l - 1) + k)$th column of $H_{SD}^i \in \mathbb{C}^{N_d \times N_s F}$, which is the channel matrix between S and D. The elements of $h_{SR}^i$ and $H_{SD}^i$ are distributed with $\mathcal{CN}(0, \Omega_{sr})$ and $\mathcal{CN}(0, \Omega_{sd})$, respectively. Moreover, $n_{SR}^i$ and $n_{SD}^i \in \mathbb{C}^{N_s \times 1}$ stand for additive white Gaussian noise (AWGN) sample at $R_i$ and AWGN vector at D, respectively. $n_{SR}^i$ and the elements of $n_{SD}^i$ are distributed with $\mathcal{CN}(0, N_0)$.

At the second stage, S suspends its signal transmission, while the relays that are able to decode correctly the active transmit antenna index and the MAP index, activate one of their transmit antennas and arrange the ON/OFF status of their RF mirrors, similar to S, corresponding to the decoded information. Moreover, the relays that are not able to decode the information amplify their received signals and forward them to D. Since each relay uses an orthogonal channel for its transmission, interrelay interference is assumed to be zero. Hence, a total of $M + 1$ orthogonal channels are required for the transmission. If $R_i$ decodes its received signal faultlessly, the received signal at D from $R_i$ can be written as

$$y_{RF}^{R_i, D} = \sqrt{E_R} h_{l,k}^{R_i, D} + n_{R_i, D}$$

(4)

where $E_R$ is the average energy of the transmitted signal from $R_i$, $h_{l,k}^{R_i, D} \in \mathbb{C}^{N_d \times 1}$ denotes the channel fading vector between $R_i$ and D corresponding to $l$th transmit antenna and $k$th MAP of $R_i$, $h_{l,k}^{R_i, D}$ is the $(F(l - 1) + k)$th column of $H_{RD}^i$, which is the channel matrix between $R_i$ and D with dimensions $N_d \times N_F$. The elements of $H_{RD}^i$ are distributed with $\mathcal{CN}(0, \Omega_{rd})$. $n_{R_i, D} \in \mathbb{C}^{N_d \times 1}$ is the AWGN sample vector at D whose elements are distributed with $\mathcal{CN}(0, N_0)$.

On the other hand, if $R_i$ decodes its received signal erroneously, it amplifies its received signal and forwards it to D. In that case, the received signal at D from $R_i$ can be written as

$$y_{AF}^{R_i, D} = A \sqrt{E_R} y_{SR}^{R_i, D} h_{l,k}^{R_i, D} + n_{R_i, D}$$

(5)

where $A = \frac{1}{\sqrt{E_{SR} + E_{AF}}} h_{l,k}^{R_i, D}$ is a fixed-gain amplification factor. $h_{l,k}^{R_i, D} \in \mathbb{C}^{N_d \times 1}$ is the vector of channel fading coefficients between $R_i$ and D corresponding to a predetermined transmit antenna and MAP of $R_i$, which is known by D and does not carry any information. After applying noise normalization as in [21], (5) can be rewritten as

$$\hat{y}_{AF}^{R_i, D} = G h_{l,k}^{R_i, D} + n_{R_i, D}$$

(6)

where $G = \frac{\sqrt{E_{SR}} h_{l,k}^{R_i, D}}{\sqrt{E_{SR} + E_{AF}}} h_{l,k}^{R_i, D}$ and $n_{R_i, D} \in \mathbb{C}^{N_d \times 1}$ is the normalized AWGN sample vector at D, which has the same statistical properties with $n_{R_i, D}$. We define the decoding sets $C$ and $E$ as the set of relays that decode the active transmit antenna and MAP indices correctly and erroneously, respectively. In other words, if the $i$th relay decodes its received signal correctly, then it is assigned to the set $C$ and vice versa. Therefore, the decision rule for the source antenna and MAP index based on the ML principle can be expressed as

$$[\hat{i}, \hat{k}] = \arg \min_{1 \leq i \leq N_s, 1 \leq j \leq F} \left\{ \left\| y_{SD}^i - \sqrt{E_S} h_{l,k}^i \right\|^2 + \left\| y_{AF}^{R_i, D} - G h_{l,k}^{R_i, D} \right\|^2 \right. + \sum_{i \in C} \left\| y_{DF}^{R_i, D} - \sqrt{E_R} h_{l,k}^{R_i, D} \right\|^2 + \left. \left\| \hat{y}_{AF}^{R_i, D} - G h_{l,k}^{R_i, D} \right\|^2 \right\}.$$  

(7)
the relays decode correctly or erroneously, cyclic redundancy check (CRC) bits are employed at S during transmission as in [17].

B. PEP Analysis

In this section, we analyze the PEP performance of SSK-MBM with HR. As \( I = (F(l - 1) + k) \), denoting the index of the channel fading coefficients corresponding to incoming data bits, the PEP of the proposed scheme, \( P(I \rightarrow \hat{I}) \), can be defined as the probability that \( I \) is erroneously detected as \( \hat{I} \). Such an error event occurs when the transmit antenna index \( l \) and/or MAP index \( k \) is erroneously detected as \( l \) and \( k \), respectively. Considering (7), the conditional PEP expression can be written as [20], [21]

\[
P(I \rightarrow \hat{I} | h^{SR}, H^{SD}, H^{R}, D) = Q \left( \sqrt{\gamma_{SD} + \sum_{i \in C} \gamma_{DF}^{R,D} + \sum_{i \in E} \gamma_{AF}^{SR,D}} \right)
\]

where \( \gamma_{SD} = \frac{E_s \| h_{SR,i}^{D} - h_{SR,i}^{P} \|^2}{N_0} \) and \( \gamma_{DF}^{R,D} = \frac{E_k \| h_{R,i}^{D} - h_{R,i}^{K} \|^2}{N_0} \).

Moreover, \( \gamma_{AF}^{SR,D} = \frac{E_s \| h_{SR,i}^{R} - h_{SR,i}^{P} \|^2}{N_0} \), where \( \gamma_{R,D} = \frac{E_k \| h_{R,i}^{K} \|^2}{N_0} \). Here, \( Q(\cdot) \) denotes the Gaussian Q-function [27, Eq. (4.1)].

In our proposed system, as a cooperative HR scheme is employed in addition to the direct link between S and D, the received signal at D contains the signals transmitted from S and the relays, which employ either AF or DF principle. The probability that \( i \)th relay decodes its received signal erroneously, in other words, \( i \)th relay is the element of the set \( E \), can be given as \( P_E^{R} = E \left[ Q \left( \sqrt{2N_0} \right) \right] \). Since \( |h_{SR,i}^{D} - h_{SR,i}^{P}|^2 \) is exponentially distributed, the cumulative distribution function (CDF) of \( \gamma_{SR} \) can be expressed as \( F_{\gamma_{SR}}(x) = 1 - e^{-\gamma_{SR} x}, \) where \( P_S = E_S/N_0 \). Therefore, \( P_E^{R} \) can be calculated with the help of [27, Eq. (5.6)] as

\[
P_E^{R} = \frac{1}{2} \left( 1 - \sqrt{\frac{P_{S} \Omega_{SR}/2}{1 + P_{S} \Omega_{SR}/2}} \right).
\]

On the other hand, the probability that the \( i \)th relay is the element of the set \( C \) can be simply written as \( P_E^{C} = 1 - P_E^{R} \).

The probability that \( v \) out of \( M \) relays are the elements of the correct detection set \( C \) can be given as \( \sum_{v=0}^{M} M! (P_E^{R})^{M-v} (P_C)^v \). Note that \( v = 0 \) corresponds to the case that all relays decode their received signal erroneously and employ AF principle. By considering such probability and averaging (8) over \( h^{SR}, H^{SD}, \) and \( H^{R,D} \), the unconditional PEP can be expressed as

\[
P(I \rightarrow \hat{I}) = \sum_{v=0}^{M} \binom{M}{v} (P_E^{R})^{M-v} (P_C)^v 
\]

\[
\times E \left[ Q \left( \sqrt{\gamma_{SD} + \sum_{i \in C} \gamma_{DF}^{R,D} + \sum_{i \in E} \gamma_{AF}^{SR,D}} \right) \right].
\]

Note that \( |C| = v \), where \( |C| \) is the cardinality of the set \( C \), and similarly \( |E| = M - v \).

Using the moment generation function (MGF) approach [27], (10) can be obtained as

\[
P(I \rightarrow \hat{I}) = \sum_{v=0}^{M} \binom{M}{v} (P_E^{R})^{M-v} (P_C)^v \frac{1}{I}
\]

\[
\times \int_{0}^{\pi/2} M_{1,0}^\text{SD} \left( \frac{1}{2 \sin^2 \theta} \right) \left[ M_{\text{DF}}^{SR,D} \left( \frac{1}{2 \sin^2 \theta} \right) \right]^v \right)
\]

\[
\times \left[ M_{\text{AF}}^{SR,D} \left( \frac{1}{2 \sin^2 \theta} \right) \right]^{M-v} \right) d\theta
\]

where \( M_{I,(\cdot)} \) is the MGF of the random variable \( Y \).

Since \( |h_{SR,i}^{D} - h_{SR,i}^{P}|^2 \) is a chi-square random variable with \( 2N_d \) degrees of freedom, the probability density function of \( \gamma_{SD} \) can be expressed as

\[
f_{\gamma_{SD}}(x) = \frac{x^{N_d-1} e^{-\gamma_{SD} x}}{(P_S \Omega_{SD})^{N_d} \Gamma(N_d)} \]

\[
\Gamma(\cdot) \text{ is the Gamma function [28, Eq. (8.310.1)]. Then, the MGF of } \gamma_{SD} \text{ can be given as}
\]

\[
M_{\gamma_{SD}}(s) = \int_{0}^{\infty} e^{-sx} f_{\gamma_{SD}}(x) dx = \left( \frac{1}{P_S \Omega_{SD} s + 1} \right)^{N_d}
\]

\[
\text{Analogously, the MGF of } \gamma_{DF}^{R,D} \text{ can be expressed as}
\]

\[
M_{\gamma_{DF}^{R,D}}(s) = \left( \frac{1}{P_P \Omega_{DF} s + 1} \right)^{N_d}
\]

where \( P_R = E_R/N_0 \). Using mathematical steps similar to the derivation of [21, Eqs. (21) and (24)], the CDF and MGF of \( \gamma_{SR,D} \) can be expressed, respectively, as

\[
F_{\gamma_{SR,D}}(x) = 1 - 2(\delta x) x \frac{e^{-\gamma_{SR,D} x}}{\Gamma(N_d)} K_{N_d}(2 \sqrt{\delta x})
\]

\[
M_{\gamma_{SR,D}}(s) = 1 - sN_d \delta^{N_d+1} \exp \left( \frac{\delta/2}{s + 1/P_S \Omega_{SR}} \right)
\]

\[
\times (s + 1/P_S \Omega_{SR})^{N_d+1} W_{N_d+1} \left( \delta/2 \right) \frac{1}{s + 1/P_S \Omega_{SR}}
\]

where \( \delta = P_R \Omega_{DF} (P_R \Omega_{SR}) \), and \( K_u(\cdot) \) and \( W_{m,n}(\cdot) \) are the \( u \)-th order modified Bessel function of the second kind [28, Eq. (8.432.1)] and the Whittaker function [28, Eq. (9.222.2)], respectively.

By substituting (13), (14), and (16) into (11) and calculating the integral numerically, the PEP of the proposed scheme can be obtained. Note that, to the best of authors’ knowledge, the integral given in (11) does not have a closed-form solution. Note that the PEP is equivalent to ABEP when \( N_f = 2 \) for SSK-MBM systems. We perform approximate ABEP analysis in Section II-D for \( N_f > 2 \).

C. Asymptotic Error Probability Analysis

In this section, we analyze the asymptotic error performance of SSK-MBM with HR. In this regard, we investigate the
analytical results derived in the previous sections for high signal-to-noise ratio (SNR) values.

The MGFs of $\gamma_{SRD}$ and $\gamma_{SRD}^{DF}$ can be approximated at high SNR values, respectively, as [21]

$$M_{\gamma_{SRD}}(s) \approx \left( \frac{1}{P_{r}\Omega_{sr}d_{s}} \right)^{N_d} \tag{17}$$

$$M_{\gamma_{SRD}^{DF}}(s) \approx \left( \frac{1}{P_{r}\Omega_{sr},d_{d}} \right)^{N_d} \tag{18}$$

Moreover, using [28, Eq. (1.211)] and [28, Eq. (8.446)], the CDF of $\gamma_{SRD}^{AF}$ can be expressed for high SNR values as

$$F_{\gamma_{SRD}^{AF}}(x) \approx x \left[ \frac{1}{P_{r}^{2}\Omega_{sr}} + \frac{\beta}{P_{r}\Omega_{sr},d_{d}(N_d - 1)} \right]. \tag{19}$$

Note that (19) is valid for $N_d > 1$. Using (19), the MGF of $\gamma_{SRD}^{AF}$ can be obtained for high SNR values and $N_d > 1$ as

$$M_{\gamma_{SRD}^{AF}}(s) \approx \frac{1}{s} \left[ \frac{1}{P_{r}^{2}\Omega_{sr}} + \frac{\beta}{P_{r}\Omega_{sr},d_{d}(N_d - 1)} \right]. \tag{20}$$

By substituting (17), (18), and (20) into (11), an asymptotic PEP expression of the proposed system can be obtained. When (17), (18), and (20) are substituted into (11), it is easy to observe that the exponential power of $s$ in such a formula, which determines the diversity order of the system, is $N_d + N_d v + M - v$. Since all relays start to decode their received signal correctly for higher values of SNR, $v$ converges to $M$ as SNR increases. Hence, the asymptotic diversity order of the system can be given as $d = N_d + N_d M$.

D. Approximate ABEP Analysis

The Gaussian $Q$-function can be upper bounded by $Q(x) \leq \frac{1}{2} \exp(-\frac{x^2}{2})$ [29]. Using this upper bound as in [3], we obtain the following upper bound:

$$P_{E}^{R} \leq \frac{1}{2P_{r}^{2}\Omega_{sr}} \tag{21}$$

Using (21), an upper bound on the ABEP of S–R, link is given by the well-known union bound as

$$P_{E}^{R} = \frac{1}{N_{s}F \log_{2}(N_{s}F)} \sum_{I=1}^{M} \sum_{I=1}^{N_{s}F} P_{E}^{R} N(I, \hat{I}) \tag{22}$$

where $N(I, \hat{I})$ denotes the number of bits in error for the corresponding pairwise error event.

On the other hand, by substituting (22) into (11) and using the well-known union bound, the ABEP of the proposed system can be approximated as

$$P_{b} \approx \frac{1}{N_{s}F \log_{2}(N_{s}F)} \sum_{I=1}^{M} \sum_{I=1}^{N_{s}F} P(I \rightarrow \hat{I}) N(I, \hat{I}) \tag{23}$$

III. SSK-MBM With HRS

A. System Model

In this section, we consider a cooperative SSK-MBM scheme with HRS, in which a relay selection technique is applied to SSK-MBM with HR in order to increase the data rate by performing the data transmission over only one (selected) relay. Hence, only two orthogonal channels, for the S–D link and for the selected relay-D link, are required for the transmission. As in the SSK-MBM with HR scheme, transmission occurs in a two-stage protocol. At the first stage, S transmits its SSK-MBM signal to the relays and D. The received signal at R, and signal vector at D from S can be written as (1) and (2), respectively. Then, each relay is assigned to either DF relaying set $C$ or AF relaying set $E$ depending on its detection result.

At the second stage, S and the unselected relays remain silent and the selected relay either amplifies or decodes its received signal and forward it to D. Here, an HRS scheme is applied over DF and AF relays as in [17]. Let us denote the selected relay index by $\lambda$. If the selected relay is in the set $C$, the received signal vector at D from the selected relay $R_{\lambda}$ can be written as in (4) by changing the index $i$ with $\lambda$ or if $\lambda \in E$, the received signal vector at D from the selected relay $R_{\lambda}$ can be written as in (6) by changing the index $i$ with $\lambda$.

Relay selection is performed considering the channel fading coefficients between $S$, $R_{i}$, and $D$. Considering the PEP expression in (11), D selects the relay, which maximizes the worst case PEP of the relaying link. In other words, the index of the selected relay is determined as

$$\lambda = \arg \max_{i,j \in \{1, \ldots, M\}, i \neq j} \left\{ \gamma_{SRD}^{DF} \right\} \tag{24}$$

Note that the index $j$ stands for the $j$th relay and the indices $i$ and $j$ are determined according to the relay’s CRC checking result. Therefore, $i$ and $j$ can be the element of either the set $C$ or $E$.

B. PEP Analysis

Considering (10), the PEP of SSK-MBM with HRS can be given as

$$P(I \rightarrow \hat{I}) = \sum_{v=0}^{M} \left( M \atop v \right) \left( P_{R}^{E} \right)^{M-v} \left( P_{R}^{R} \right)^{v} \times E \left[ Q \left( \gamma_{\text{SD}} + \gamma_{\text{SRD}}^{\text{max},v} \right) \right] \tag{25}$$

where $\gamma_{\text{SRD}}^{\text{max},v}$ corresponds to the maximum of the worst case PEP expressions belonging to the S–R–D links when $v$ out of $M$ relays decode their received signal correctly and $M - v$ out of $M$ relays decode their received signal erroneously.

On the other hand, considering the constellation diagram of SSK-MBM for the S–R and R–D links, the total number of different squared Euclidean distances in the diagram is equal to $T = \frac{v^2}{2}$ and these distances are statistically dependent. Hence, it may not be possible to obtain distribution functions for $\gamma_{\text{SRD}}^{\text{max},v}$. For the sake of obtaining approximate statistics for the selected relay, we assume that $T$
Euclidean distances within the PEP expression are independent and these distances follow chi-square distribution as in SSK-MBM with HR. In that case, we define such a random variable as $\gamma_{\text{SRD}}^\max = \max_{i,j \in \{1, \ldots, M\}} \left\{ \frac{D_{i,j}}{R_{i,j}} \right\}$, where $\gamma_{\text{DF},i,j}^\max$ and $\gamma_{\text{AF},i,j}^\max$ are defined as $\gamma_{\text{DF},i,j}$ and $\gamma_{\text{AF},i,j}$, respectively; however, unlike $\gamma_{\text{DF},i,j}$ and $\gamma_{\text{AF},i,j}$, we assume that $\left( N, F \right)$ Euclidean distances within $\gamma_{\text{DF},i,j}$ and $\gamma_{\text{AF},i,j}$ are independent. Therefore, using the MGF approach [27], (25) can be upper bounded as

$$P_s(I \rightarrow \hat{I}) \leq \sum_{v=0}^{M} \left( \frac{M!}{v!} \right) \left( \frac{P_{bE}}{P_{bC}} \right)^v \left( \frac{1}{2} \right)^{v/2} \left( \frac{1}{\sin^2 \theta} \right) M_{\gamma_{\text{SRD}}^\max} \left( \frac{1}{2} \sin^2 \theta \right) d\theta.$$  

(26)

Since the channel fading coefficients corresponding to each relay are independent and identically distributed, the CDF of $\gamma_{\text{SRD}}^\max$ can be expressed, with the help of order statistics, as

$$F_{\gamma_{\text{SRD}}^\max} (x) = \left[ F_{\gamma_{\text{DF},i,j}^\max} (x) \right]^v = \left[ 1 - \left( 1 - F_{\gamma_{\text{DF},i,j}^\max} (x) \right)^v \right].$$

(27)

where $F_{\gamma_{\text{DF},i,j}^\max} (x)$ is the CDF of $\gamma_{\text{DF},i,j}$ and it can be calculated, with the help of [28, Eq. (3.381.1)], as

$$F_{\gamma_{\text{DF},i,j}^\max} (x) = \int_0^x f_{\gamma_{\text{DF},i,j}^\max} (\gamma) d\gamma = \int_0^x \gamma_{N_1,i,j}^{N_1,i,j} e^{-\gamma_{N_1,i,j}} \left( \frac{P_{bE}}{P_{bC}} \right)^N d\gamma,$$

(28)

where $\gamma_{..}$ is the incomplete lower Gamma function [28, Eq. (8.350.1)]. By substituting (15) and (28) into (27), the CDF of $\gamma_{\text{SRD}}^\max$ can be obtained. Furthermore, the MGF of $\gamma_{\text{SRD}}^\max$ can be approximated as

$$M_{\gamma_{\text{SRD}}^\max} (s) = 8 \int_0^\infty e^{-sx} F_{\gamma_{\text{SRD}}^\max} (x) dx.$$  

(29)

Unfortunately, to the best of our knowledge, there is no closed-form solution for the integral in (29).

On the other hand, by substituting (13) and (29) into (26) and evaluating the integral numerically, an upper-bound expression on the PEP of SSK-MBM with HRS can be obtained. Note that the equation holds for $N, F = 2$, i.e., $T = 1$.

C. Asymptotic Error Probability Analysis

For $N, F = 2$, (26) holds and $\gamma_{\text{SRD}}^\max = \gamma_{\text{SRD}}^\max$. Hence, we find it convenient to use $\gamma_{\text{SRD}}^\max$ instead of $\gamma_{\text{SRD}}^\max$. Moreover, using [28, Eq. (1.211)] and [28, Eq. (8.446)], the CDF of $\gamma_{\text{SRD}}^\max$ can be approximated for high SNR values and $N_d > 1$ as

$$F_{\gamma_{\text{SRD}}^\max} (x) \approx \left[ \frac{(x/P_{b\Omega_d})^{N_d}}{N_d!} \right]^v \left[ \frac{1}{P_{b\Omega_d} + P_{b\Omega_d} \beta (N_d - 1)} \right]^{M-v}.$$  

(30)

Using (30), the MGF of $\gamma_{\text{SRD}}^\max$ can be obtained for high SNR values and $N_d > 1$ as

$$M_{\gamma_{\text{SRD}}^\max} (s) \approx \frac{\Gamma(M + N_d v - v + 1)}{s^M + N_d v - v} \left[ \frac{(1/P_{b\Omega_d})^{N_d}}{N_d!} \right]^v \left[ \frac{1}{P_{b\Omega_d} + P_{b\Omega_d} \beta (N_d - 1)} \right]^{M-v}.$$  

(31)

On the other hand, the MGF of $\gamma_{\text{SRD}}^\max$ can be approximated as in (17). Moreover, the asymptotic PEP expression of SSK-MBM with HRS can be obtained by substituting (17) and (31) into (25). It is easy to observe that the exponential power of $s$ in such a formula, which determines the diversity order of the system, is $N_d + N_d v + M - v$. Since increasing SNR would prevent the relay from erroneous detection as in the previous scheme, $v$ converges to $M$ as the SNR increases. Hence, the asymptotic diversity order of the system can be given as $d_k = N_d + N_d M$.

D. Approximate ABEP Analysis

The ABEP of S-$$R_i$$ link can be upper bounded as in (22). Moreover, by substituting (22) into (26) and using the well-known union bound, the ABEP of SSK-MBM with HRS can be approximated as

$$P_s \approx \frac{1}{N_s F \log_2 (N_s F)} \sum_{I=1}^{N_s} \sum_{I=1}^{F} P_s(I \rightarrow \hat{I}) N(I, \hat{I}).$$  

(32)

IV. SIMULATION RESULTS AND COMPARISONS

In this section, the theoretical expressions derived in the previous sections are validated through Monte Carlo simulations and the proposed schemes are compared with conventional cooperative SSK-MBM as well as conventional single-input–multiple-output–quadrature amplitude modulation (SIMO-QAM) schemes. Further BER comparisons are performed with the SM schemes. The results in this section are provided as a function of $E_T/N_0$, where $E_T = E_S + M E_R$. Note that since only one relay is activated, $M = 1$ and $E_T = E_S + E_R$ for the schemes with relay selection.

In Fig. 2(a) and (b), we compare our computer simulation results with the theoretical PEP curves obtained in Sections II-B and III-B for SSK-MBM with HR and HRS, respectively, where we consider $N, F = 2$, $M \in \{1, 2, 3, 4\}$, and $N_d = 2$. Note that $N, F = 2$ means that the transmission scheme is either an SSK scheme with two transmit antennas ($N_S = 2, F = 1$) or an MBM scheme with one RF mirror ($N_S = 1, F = 2$). Fig. 2 clearly demonstrates that the simulation results match with the analytical PEP and asymptotic diversity order results given in the previous sections and the
BER performance of the proposed schemes is improved when the number of available relays $M$ increases.

In Fig. 3, we investigate the BER performance of the proposed schemes in order to show the validity of the approximated ABEP expressions obtained for relatively larger values of $N_s$ and $F$ in Sections II-D and III-D. In Fig. 3, the curves of SSK-MBM with HR are given for $[N_s, F] \in \{2, 2\}$, $M = 2$, and $N_d = 2$; however, the curves of SSK-MBM with HRS are given for $[N_s, F] \in \{2, 2\}$, $M = 2$, and $N_d = 4$. As seen from Fig. 3, the derived approximate ABEP expressions are considerably accurate for especially higher SNR values.

Fig. 4(a) and (b) compares the BER performance of the proposed schemes for the different values of $[N_s, F]$ and the different number of available relays $M$, respectively. The BER curves in Fig. 4(a) are given for $[N_s, F] \in \{1, 2\}$, $M = 2$, and $N_d = 3$. On the other hand, the BER curves in Fig. 4(b) are given for $[N_s, F] = \{2, 2\}$, $M \in \{2, 4, 6\}$, and $N_d = 4$. As seen from Fig. 4(a), SSK-MBM with HR outperforms SSK-MBM with HRS by almost preserving the same SNR gap as the difference of SNRs to achieve the same target BER. However, the SNR gap between SSK-MBM with HR and HRS schemes increases when the number of available relays increases and the effectiveness of SSK-MBM with HR becomes more evident when the number of available relays increases as seen from Fig. 4(b). Such a BER improvement of SSK-MBM with HR compared to SSK-MBM with HRS is obtained by sacrificing the data rate since it activates multiple relays using orthogonal subchannels. As a result, we observe an inevitable tradeoff between the error performance and data rate by SSK-MBM with HR compared to SSK-MBM with HRS. However, since SSK-MBM with HRS provides the same diversity order as SSK-MBM with HR compared to SSK-MBM with HRS is obtained by sacrificing the data rate since it activates multiple relays using orthogonal subchannels. As a result, we observe an inevitable tradeoff between the error performance and data rate by SSK-MBM with HRS. However, since SSK-MBM with HRS provides the same diversity order as SSK-MBM with HR, it would be a more effective solution for high SNR values. On the other hand, SSK-MBM with HR would be a better choice for the cases that the data rate is less important than BER and the SNR gap with increasing data rate. Note that we define the SNR gap as the difference of SNRs to achieve the same target BER.

We compare the proposed HR scheme with SSK-MBM with conventional DF relaying (DFR) and AF relaying (AFR) for $[N_s, F] \in \{2, 2\}$, $M = 3$, and $N_d = 1$ in Fig. 5(a) and for $[N_s, F] = \{2, 2\}$, $M \in \{1, 3, 5\}$, and $N_d = 2$ in Fig. 5(b). In the reference DFR scheme, we consider that the relays, which

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**Fig. 2.** BER performance curves for $N_s = 2$, $M \in \{1, 2, 3, 4\}$, and $N_d = 2$. (a) SSK-MBM with HR. (b) SSK-MBM with HRS.

**Fig. 3.** Comparison of the approximated ABEP curves with the Monte Carlo simulation results for SSK-MBM with HR and $[N_s, F] \in \{2, 2\}$, $M = 2$, $N_d = 2$; SSK-MBM with HRS and $[N_s, F] \in \{2, 2\}$, $M = 2$, $N_d = 4$.

**Fig. 4.** BER performance comparisons. (a) SSK-MBM with HR and HRS for $[N_s, F] \in \{1, 2\}$, $M = 2$, $N_d = 3$. (b) SSK-MBM with HR and HRS for $[N_s, F] = \{2, 2\}$, $M \in \{2, 4, 6\}$, $N_d = 4$.
decode the source signal correctly, take part in the transmission. On the other hand, the conventional AFR is applied in the reference AFR scheme. As seen from Fig. 5(a) and (b), SSK-MBM with HR considerably outperforms both of the reference schemes in terms of BER performance at any data rate and number of relays. It is also important to note that SSK-MBM with DFR outperforms SSK-MBM with AFR for \( [N_s, F] = [(2, 2)] \); however, when we increase the data rate, e.g., the \( [N_s, F] = [(4, 16)] \) case, the relays in the DFR scheme begin to decode their received signals erroneously and the AFR becomes a more effective relaying technique than DFR in terms of the BER performance.

We compare the proposed HRS scheme with SSK-MBM with DF relay selection (DFRS) for \( [N_s, F] = [(1, 2), (4, 4)] \), \( M = 3, N_d = 2 \) in Fig. 6(a) and provide BER comparisons of the proposed HRS scheme with SSK-MBM with AFR relay selection (AFRS) for \( [N_s, F] = [(1, 2)] \), \( M \in \{2, 3, 4\}, N_d = 2 \) in Fig. 6(b). In the reference DFRS scheme, we consider that one of the relays, which decodes the source signal correctly, is selected with respect to PEP of the R–D link. On the other hand, best relay selection in [21] is applied in the reference AFRS scheme. As seen from Fig. 6(a), the DFRS scheme outperforms the proposed HRS scheme for \( [N_s, F] = [(1, 2)] \); however, the effectiveness of the proposed scheme against the DFRS scheme is observed at higher data rates, i.e., \( [N_s, F] = [(4, 4)] \). It is obvious from Fig. 6(b) that the proposed HRS scheme provides a superior BER performance compared to the AFRS scheme and such superiority of the proposed scheme is preserved when we increase the number of available relays.

In Fig. 7, the proposed schemes are compared with conventional SIMO-QAM schemes with HR and HRS, in which the single-antenna source and relays are capable of transmitting Q-QAM symbols. More explicitly, we compare SSK-MBM with HR scheme and SIMO-QAM with HR for \( [N_s, F] \in \{[2, 2], [2, 4]\}, Q \in \{4, 8\}, M = 3, N_d = 1 \) in Fig. 7(a) as well as SSK-MBM with HRS scheme and SIMO-QAM with HRS for \( [N_s, F] = [(4, 16)], Q = 64, M = 3, N_d \in \{1, 4\} \) in Fig. 7(b). As seen from Fig. 7(a), the conventional SIMO-QAM with HR outperforms the proposed SSK-MBM with HR for \( [N_s, F] = [2, 2] \), \( Q = 4 \); however, when the data rate increases, the superiority of the proposed scheme against the con-
In this paper, we have proposed the concepts of cooperative SSK-MBM with HR and HRS, which ensure full cooperative diversity and exploit the advantages of both the SSK and MBM systems. The proposed SSK-MBM schemes can be considered as generalization of either conventional SSK schemes to MAP-based channel state domain or MBM schemes to antenna-based channel state domain. The proposed schemes can also be regarded as general relaying schemes since they include both AF and DF relaying. It has been revealed via comprehensive computer simulations that the proposed SSK-MBM schemes can provide significant improvements in BER performance compared to conventional relaying as well as conventional SIMO-QAM and SM schemes. It is also shown that there is an interesting tradeoff between SSK-MBM with HR and HRS in terms of error performance and data rate as in conventional systems. In this regard, the proposed SSK-MBM systems can be considered as potential candidates for next-generation ultra-reliable communication systems and the proposed SSK-MBM with HRS can also be considered as a candidate for low-latency communication with its reduced complexity and energy consumption. Furthermore, there exist many interesting open research problems including low complexity detection as well as performance analysis in more general fading channels for SSK-MBM with HR and HRS schemes. A combination of the proposed schemes with OFDM or pulse-shaping transmissions is also an interesting open research problem and can be investigated as a future work.

**REFERENCES**


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1Such an expression is used by Som and Chockalingam [22] to define the proposed relay selection technique.

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**Fig. 8. BER performance comparisons. (a) SSK-MBM with HR and SM with HR for \( N_s = 4, F, Q \in \{8, 64\}, M = 3 \), and \( N_d = 4 \). (b) SSK-MBM with HRS and SSK with BRS-SC [22] for \( N_s = 2, F = 1, M \in \{2, 4\}, \text{and } N_d = 4 \).**

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**V. CONCLUSION**

In this paper, we have proposed the concepts of cooperative SSK-MBM with HR and HRS, which ensure full cooperative diversity and exploit the advantages of both the SSK and MBM systems. The proposed SSK-MBM schemes can be considered as generalization of either conventional SSK schemes to MAP-based channel state domain or MBM schemes to antenna-based channel state domain. The proposed schemes can also be regarded as general relaying schemes since they include both AF and DF relaying. It has been revealed via comprehensive computer simulations that the proposed SSK-MBM schemes can provide significant improvements in BER performance compared to conventional relaying as well as conventional SIMO-QAM and SM schemes. It is also shown that there is an interesting tradeoff between SSK-MBM with HR and HRS in terms of error performance and data rate as in conventional systems. In this regard, the proposed SSK-MBM systems can be considered as potential candidates for next-generation ultra-reliable communication systems and the proposed SSK-MBM with HRS can also be considered as a candidate for low-latency communication with its reduced complexity and energy consumption. Furthermore, there exist many interesting open research problems including low complexity detection as well as performance analysis in more general fading channels for SSK-MBM with HR and HRS schemes. A combination of the proposed schemes with OFDM or pulse-shaping transmissions is also an interesting open research problem and can be investigated as a future work.

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