Molecular Index Modulation With Space-Time Equalization

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***Abstract*—Molecular multiple-input-multiple-output (MIMO) techniques are proposed in the literature to improve the overall throughput of molecular communication via diffusion (MCvD) systems. Among these methods, molecular index modulation (molecular-IM) schemes are recently introduced as novel molecular MIMO approaches to provide significant improvements in terms of communication efficiency. However, despite molecular-IM's strong combating capability against inter-symbol interference (ISI) and inter-link interference (ILI), they are still prone to them due to the characteristics of a molecular MIMO MCvD channel. With the goal of further mitigating ISI and ILI, this letter introduces memory-assisted, linear time and space-time equalization methods for molecular-IM schemes. It is shown that the proposed equalization methods yield significant improvements in error performance while retaining low decoding complexity due to the linear combining.**

***Index Terms*—Molecular communication, index modulation, molecular MIMO, space-time equalization.**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) approaches are proposed for molecular communication via diffusion (MCvD) systems to increase communication efficiency [1]. Among molecular MIMO schemes, including space-time block coding schemes [2] and spatial multiplexing (SMUX) [3], molecular index modulation (molecular-IM) raises interest due to its efficient combating capability against both inter-symbol interference (ISI) and inter-link interference (ILI) [4]. Overall, molecular-IM can be defined as a member of the molecular MIMO modulation family that involves encoding information in both antenna indices (spatial constellations) and signal constellations. Recently proposed molecular-IM methods include a scheme where spatial constellations are the only way to convey information [4], and other schemes where they are used in conjunction to messenger molecule (MM) type, concentration, and temporal position-based signal constellations [4]–[6].

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Even though molecular-IM yields promising results in terms of ISI and ILI mitigation, these sources of interference are still in effect on molecular-IM schemes due to the very nature of the MIMO MCvD channel. For molecular MIMO systems that do not use molecular-IM, there are several works that propose schemes to combat these adverse effects. For SMUX-based molecular MIMO, [3] considers detection schemes both with and without channel impulse response at the receiver. Furthermore, [7] proposes decision feedback type equalizers with zero forcing and minimum mean squared error criteria for SMUX-based MIMO MCvD.

Inspired by the promising results of early equalization studies by [7] on SMUX-based schemes and known feasibility benefits of linear equalizers on micro- and nano-scale single-input single-output MCvD systems [8], we propose a linear equalization framework for reducing the spatial constellation errors in molecular-IM schemes. Categorizing the proposed equalization schemes as time equalizers (TE) and space-time equalizers (STE), we tackle both ISI (TE) and ISI&ILI (STE) issues. We derive analytical cost functions with the goal of minimizing spatial constellation symbol error rates (SER), and our results show that molecular-IM can achieve promising error performances by combining linear STEs and simple detectors. Other than their simplicity, another advantage of the proposed schemes is that they do not work on a decision feedback principle, which inhibits adverse error propagation issues and provides usability in channels with harsher ISI/ILI. Overall, the proposed approaches mitigate spatial constellation error rates for molecular-IM and can be used in conjunction with any aforementioned molecular-IM scheme.

II. System Model

The system model of this letter is adopted from [4], where a molecular MIMO system with n_{TX} transmitter (TX) and n_{RX} receiver (RX) antennas is considered. TX antennas are considered to be point sources and RX antennas are absorbing spheres with radii r_r. We consider a uniform circular array (UCA) topology with $n_{TX} = n_{RX} = 8$, where corresponding TX and RX antennas are spatially aligned. The radii of the TX and RX UCAs are denoted as r_{UCA}, and the distance between TX and RX bodies is d_{TX-RX}. Other than their respective antennas, the TX body is transparent to the MMs, whilst the RX body is elastically reflective, as also considered in [4], [6]. D denotes the diffusion coefficient of the utilized MM. Fig. 1 presents the considered topology.

The most basic molecular-IM modulation, molecular space shift keying (MSSK), utilizes the antenna index as the only way to convey information [4]. In MSSK, the TX maps the information bit stream into blocks of $\log_2(n_{TX})$, and arranges the activated antenna index sequence, $\mathbf{x}$, accordingly.

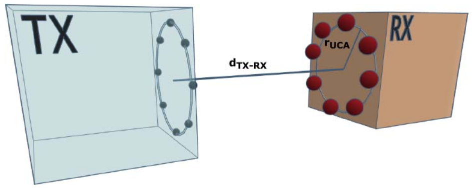

Fig. 1. Considered system model for $n_{TX} = n_{RX} = 8$.

In the beginning of the k^{th} symbol interval, the TX emits $M\log_2(n_{TX})$ MMs from *x*[*k*], where $x[k] \in \{1,\ldots,n_{TX}\}$ is the k^{th} element of **x** and *M* is the transmitted number of MMs (per bit) budget of the TX, respectively.

After their emissions, the MMs randomly diffuse and yield probabilistic arrivals at the RX antennas. This behavior results in an $n_{RX} \times L$ channel coefficient matrix for our topology of interest, in which each entry $h_{i,j}[m]$ defines the arrival probability at the j^{th} RX antenna of a *single* MM that is emitted from the i^{th} transmit antenna, at the m^{th} symbol duration after its release. *L* denotes the total channel memory of the system. When emitting multiple MMs, total arrival count at each antenna during each symbol interval is commonly approximated as independent binomial random variables (RV) ([3], [4], [6]), which can be further approximated as normal RVs as $R_j[k] \sim \mathcal{N}(\mu_j[k], \sigma_j^2[k])$, where

$$\mu_j[k] = \sum_{z=k-L+1}^{k} \sum_{i=1}^{n_{TX}} s_i[z]h_{i,j}[k-z+1] \tag{1}$$

and

$$\sigma_j^2[k] = \sum_{z=k-L+1}^{k} \sum_{i=1}^{n_{TX}} s_i[z]h_{i,j}[k-z+1]\big(1-h_{i,j}[k-z+1]\big). \tag{2}$$

Here, $s_i[z]$ denotes the number of MMs transmitted from the i^{th} antenna during symbol interval *z*. Note that $R_j[k]$ values constitute the received signal in a molecular MIMO setting. At the RX end, the maximum count decoder (MCD) performs

$$\hat{x}[k] = \arg \max_{j\in\{1,\ldots,n_{RX}\}} R_j[k] \tag{3}$$

to detect the activated TX antenna index at k^{th} time slot [4]. MCD is a promising detector in terms of micro and nano-scale use due to its simplicity and ease of implementation.

III. Proposed Equalization Schemes

A. Time Equalization

Even though molecular-IM provides decent performance under ISI, spatially modulated molecular MIMO systems are still prone to ISI [4]. Motivated by this fact, this subsection proposes a memory-aided linear TE that works in conjunction with the MCD. Given that the RX has access to the channel coefficients [7], the proposed scheme's main goal is to *purify* the received signal vector $\mathbf{r}[k] = \big[R_1[k] \quad R_2[k] \quad \cdots \quad R_{n_{RX}}[k]\big]$ of ISI, before feeding it into the MCD block.

The key phenomenon the proposed equalizer exploits is that earlier transmissions' effects on current ISI are reflected on the earlier received MM counts. Denoting α_v as the equalizer coefficient corresponding to the w^{th} delay element, the operation for an *L*-tap linear TE can be described as

$$R'_j[k] = R_j[k] + \sum_{v=1}^{L-1} \alpha_v R_j[k-v], \tag{4}$$

where $R'_j[k]$ is the TE output corresponding to the j^{th} received antenna. Note that, after equalization, $\mathbf{r}'[k] = \big[R'_1[k] \quad R'_2[k] \quad \cdots \quad R'_{n_{RX}}[k]\big]$ is fed to the MCD block described by (3) for detection.

Considering the significantly less contribution of larger delay elements on ISI, this letter simplifies (4) to a single-tap linear TE, simply described by

$$R'_j[k] = R_j[k] + \alpha R_j[k-1]. \tag{5}$$

Here, α represents the single-delay TE coefficient. Note that in order for the proposed TE to improve SER performance, proper selection of α is critical.

1) Finding the Optimal α Coefficient: The equalization operation in (5) is a linear combination of $R_j[k]$ and $R_j[k-1]$. Therefore, recalling $R_j[k] \sim \mathcal{N}(\mu_j[k], \sigma_j^2[k])$ and the independent arrival approximation, $R'_j[k]$ is also normally distributed with mean $\mu'_j[k] = \mu_j[k] + \alpha\mu_j[k-1]$ and variance $\sigma_j^{2\prime}[k] = \sigma_j^2[k] + \alpha^2\sigma_j^2[k-1]$. In addition, the main goal of minimizing SER can also be interpreted as maximizing the correct detection probability. According to its definition in (3), MCD detects the largest arrival count among all n_{RX} counts. Therefore, the probability of correct detection corresponds to the probability of the intended antenna's arrival count being the largest among all n_{RX} normally distributed RVs.

The signal-dependent nature of the MIMO MCvD channel necessitates averaging over all conditional error probabilities of possible activated antenna sequences to obtain the exact theoretical SER, which requires the conditional SER evaluation of all combinations in the *L*-tap channel ($x[k-L+1{:}k]$). However, even though it does not yield the exact SER value, an evaluation over a shorter memory selection of L_{eq} for feasibility purposes is found to be sufficient for the task of finding a close-to-optimal α (α_T). Furthermore, the probability of making an error on each antenna is equal due to the symmetry introduced by the UCA topology considered in this letter. Therefore, taking the antenna with index 1 as the reference for calculation (in other words, assuming $x[k] = 1$), the optimum TE coefficient that maximizes the correct detection probability (minimizes SER) can be found by

$$\alpha_T = \arg\max_{\alpha} \Big\{\mathbb{E}_{x[k-L_{eq}:k-1]}\big[P\big(R'_1[k] > \max(R'_2[k],\ldots,R'_{n_{RX}}[k])\big)\big]\Big\}. \tag{6}$$

Here, $\mathbb{E}_{x[k-L_{eq}:k-1]}[\,\cdot\,]$ represents the expectation operation over all $x[k-L_{eq}{:}k-1]$. In addition, even though (6) is written specifically for a topology where all probabilities of making an error on each antenna are equal (2×2, 4×4 square, $n\times n$ UCA, etc.), it can easily be generalized to a non-symmetrical antenna topology. We present (6) as is, since we use it as presented in this letter.

Due to the independence approximation mentioned in Section II, $P(R'_1[k] > \max(R'_2[k],\ldots,R'_{n_{RX}}[k]))$ can be written as $\prod_{j=2}^{n_{RX}} P(R'_1[k] > R'_j[k])$ as shown in [4], which can be used to transform (6) into

$$\alpha_T = \arg\max_{\alpha}\left\{\mathbb{E}_{x[k-L_{eq}:k-1]}\left[\prod_{j=2}^{n_{RX}} P\big(R_1'[k] > R_j'[k]\big)\right]\right\}. \quad (7)$$

Lastly, by relating (7) to the well-known tail distribution of the standard normal distribution (the *Q*-function) as

$$\alpha_T = \arg\max_{\alpha}\left\{\mathbb{E}_{x[k-L_{eq}:k-1]}\left[\prod_{j=2}^{n_{RX}} Q\left(\frac{\mu_j'[k]-\mu_1'[k]}{\sqrt{\sigma_j^{2\prime}[k]+\sigma_1^{2\prime}[k]}}\right)\right]\right\}, \quad (8)$$

the derivation of the optimal coefficient for the one-tap linear molecular-IM TE is complete. α_T can be numerically found by evaluating (8). Note that in order to ensure optimality, $L_{eq} = L$ should be selected and any $L_{eq} < L$ yields a suboptimal α_T. Furthermore, this approach can easily be extended to the *L*-tap equalizer defined by (4), although the numerical evaluation is expected to take a significantly longer time.

2) An Alternative Sub-Optimal Cost Function: Even though (8) theoretically finds α_T, it requires multiple *Q*-function calls for each α value, which is undesirable due to its computational complexity. Here, we propose an alternative, lower-complexity sub-optimal cost function to theoretically find α_T.

The proposed approach assumes that the majority of errors on any antenna are due to detecting one of its adjacent antennas. Furthermore, it argues on the principle that the *worst symbol sequence* in terms of SER contribution yields the dominating component of SER. Stemming from these claims, the proposed approach follows a $\max - \min$ strategy: it finds α_T that maximizes the minimum difference between the means of $R_1'[k]$ and its adjacent arrival RVs. Mathematically, the alternative method finds α_T to be

$$\alpha_T = \arg\max_{\alpha}\left\{\min_{x[k-L_{eq}:k-1]} \mu_1'[k] - \frac{\mu_2'[k]+\mu_{n_{RX}}'[k]}{2}\right\}. \quad (9)$$

Note that (9) is relatively less complex than (8), requiring only several basic operations for each evaluated α.

B. Space-Time Equalization

Similar to the argument made for ISI, ILI is still existent in an MCvD-based molecular-IM system [9]. Motivated by this fact, we extend the idea of TE to propose a linear STE for molecular-IM. Overall, (4) can be extended to represent the general form of the linear *L*-tap STE for molecular-IM as

$$R_j''[k] = \sum_{v=0}^{L-1}\sum_{w=0}^{n_{RX}-1} \alpha_{v,w} R_{(\widetilde{j+w})}[k-v], \quad (10)$$

where $(\tilde{b}) = (b-1)_{n_{RX}} + 1$, and $(\cdot)_{n_{RX}}$ denotes the modulo n_{RX} operation. Since multiplying every arrival count with the same constant does not change their comparative largeness, a normalization ensuring $\alpha_{0,0} = 1$ can be made. Furthermore, due to the spherical symmetry introduced by the UCA topology considered in this letter, $\alpha_{v,w} = \alpha_{v,n_{RX}-w}$ is valid.

In addition to the significantly less ISI contribution of larger delay elements, a claim stating that non-adjacent antennas contribute to ILI significantly less than adjacent ones can also be made. Hence, similar to the simplification of TE to (5) from (4), a simplification on (10) can also be made. When simplifying the linear STE, only the arrival counts corresponding to the current and single-delayed received symbol slots of the intended and two adjacent antennas are considered. The remaining six terms describe the simplified STE as

$$\begin{aligned} R_j''[k] = {} & \alpha_{0,0}R_j[k] + \alpha_{1,0}R_j[k-1] \\ & + \alpha_{0,1}R_{(\widetilde{j+1})}[k] + \alpha_{0,n_{RX}-1}R_{(\widetilde{j-1})}[k] \\ & + \alpha_{1,1}R_{(\widetilde{j+1})}[k-1] + \alpha_{1,n_{RX}-1}R_{(\widetilde{j-1})}[k-1]. \end{aligned} \quad (11)$$

Note that for some antenna arrangements, antennas may have different numbers of neighbors. Noting one needs to write multiple equations to describe the simplified STE for those topologies, we keep our scope within the UCA arrangement.

Recalling $\alpha_{v,w} = \alpha_{v,n_{RX}-w}$ due to symmetry in this letter and $\alpha_{0,0} = 1$, (11) can be equivalently written by naming the appropriate coefficients as α, β, and θ, as

$$\begin{aligned} R_j''[k] = {} & R_j[k] + \alpha R_j[k-1] + \beta R_{(\widetilde{j+1})}[k] \\ & + \beta R_{(\widetilde{j-1})}[k] + \theta R_{(\widetilde{j+1})}[k-1] + \theta R_{(\widetilde{j-1})}[k-1]. \end{aligned} \quad (12)$$

For example, $R_4''[k]$ can simply be found as $R_4''[k] = R_4[k] + \alpha R_4[k-1] + \beta R_5[k] + \beta R_3[k] + \theta R_5[k-1] + \theta R_3[k-1]$. Lastly, the output for the STE is fed to the MCD for detection.

In order to find the optimal α, β, and θ theoretically, a similar approach to (6) needs to be employed. Again, by assuming $x[k] = 1$ courtesy of the symmetry, the close-to-optimal coefficient triple for the STE defined by (12), $[\alpha_{ST}, \beta_{ST}, \theta_{ST}]$, can be obtain by performing

$$\begin{aligned} & [\alpha_{ST}, \beta_{ST}, \theta_{ST}] = \arg\max_{\alpha,\beta,\theta} \\ & \left\{\mathbb{E}_{x[k-L_{eq}:k-1]}\left[P\big(R_1''[k] > \max(R_2''[k],\ldots,R_{n_{RX}}''[k])\big)\right]\right\}. \end{aligned} \quad (13)$$

Even if the individual arrivals are approximated as independent, linear STE outputs corresponding to adjacent antennas (e.g., $R_4''[k]$ and $R_5''[k]$) become statistically dependent due to the STE operation defined in (11). This dependence inhibits the equivalence between the joint probability $P(R_1''[k] > \max(R_2''[k],\ldots,R_{n_{RX}}''[k]))$ and the product of marginal probabilities, and necessitates the evaluation of (13) using this joint probability. Said expression can be evaluated by averaging over the PDF of $R_1''[k]$, $f_{R_1''[k]}(r)$, as

$$\begin{aligned} & P\big(R_1''[k] > \max(R_2''[k],\ldots,R_{n_{RX}}''[k])\big) \\ & = \int_{-\infty}^{\infty} f_{R_1''[k]}(r) P\big(r > \max(R_2''[k],\ldots,R_{n_{RX}}''[k])\big)\, dr. \end{aligned} \quad (14)$$

Unfortunately, the distribution of the maximum of correlated Gaussian RVs does not have a closed form solution [10]. Even though several sampling-based numerical approaches are present in the literature ([10], [11]), these methods are computationally very challenging for a micro- or nano-scale device to perform. Therefore, we omit further analysis on the optimal coefficients for the STE, and believe the finding of alternative sub-optimal cost functions for molecular-IM linear STE is a future research direction to focus on.

IV. Numerical Results and Discussion

This section presents the SER results corresponding to 8-MSSK with and without the proposed equalization schemes. Fig. 2 is presented to discuss the effects of the one-tap TE described by (5), on SER. To generate Fig. 2, the actual

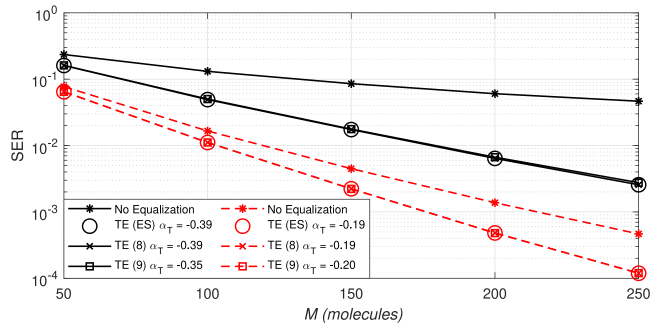

Fig. 2. SER vs. M curves for MCD-demodulated 8-MSSK with and without single-tap time equalization. $r_{UCA} = 15\,\mu\text{m}$, $d_{TX\text{–}RX} = 20\,\mu\text{m}$, $r_r = 5\,\mu\text{m}$, $D = 79.4\,\mu\text{m}^2 s{-}1$, $L = 30$, and $L_{eq} = 4$. $t_b = 0.15\,\text{s}$ for black curves, 0.25 s for red curves.

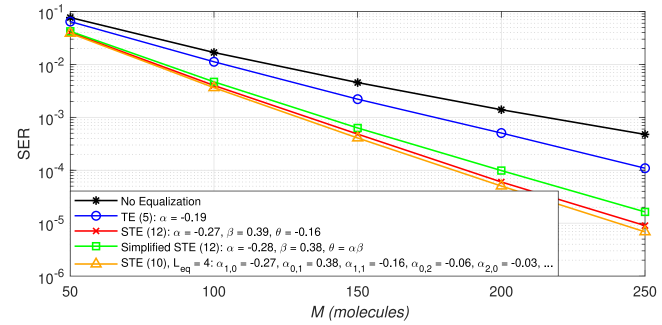

Fig. 3. SER vs. M curves for MCD-demodulated 8-MSSK with and without time and space-time equalization. $r_{UCA} = 15\,\mu\text{m}$, $d_{TX\text{–}RX} = 20\,\mu\text{m}$, $r_r = 5\,\mu\text{m}$, $D = 79.4\,\mu\text{m}^2 s{-}1$, $L = 30$, $L_{eq} = 4$, and $t_b = 0.25\,\text{s}$.

optimum value of α is found via exhaustive search (ES), and compared with the α_T values found by (8) and (9). All searches are performed on a discretized α-axis with a resolution of 0.01, for demonstration.

The results in Fig. 2 show that as predicted, TE does indeed improve SER performance. The improvement of SER is more pronounced when the bit duration t_b is lower, which is expected due to increased ISI and a higher need for TE for low t_b. Even though additional expressions and figures that relate α_T to t_b could not be provided due to space restrictions, our results show that as t_b increases, the effect of ISI decreases, mitigating the need for TE and lowering $|\alpha_T|$. Asymptotically, as $t_b \rightarrow \infty$, ISI would practically be non-existent and $\alpha_T \rightarrow 0$. In addition, since the α_T value found by (8) yields the exact α value for both $t_b = 0.15\,\text{s}$ and $0.25\,\text{s}$, one may infer that $L_{eq} = 4$ yields sufficiently close-to-optimal α values for the system topology in hand. Lastly, Fig. 2 shows an increased discrepancy between the α_T value found by (5) and the optimal α for $t_b = 0.15\,\text{s}$, even though the SER value yielded by (5) is still close to the ones of (8) and the optimal α. This behavior is mainly the result of very high ISI at $t_b = 0.15\,\text{s}$. Note that (5) assumes the errors caused by the detection of adjacent antennas dominate SER. When ISI is very high, this assumption may not hold as exemplified in [4] and [6], and causes (5) to underestimate the effect of ISI.

Fig. 3 demonstrates the improvement brought by STE over TE and the difference in performance of the STEs described by (10)-(12). In order to further simplify the search space of (12) to only two variables, a sub-optimal subset of (12) where θ is heuristically equated to $\theta = \alpha\beta$ is also presented. All equalizer coefficients in the figure are optimized using custom MATLAB algorithms, and all searches are performed on discretized axes with 0.01 steps. Note that due to extreme computation time of optimizing all $\alpha_{v,w}$ values of (10) for $n_{RX} = 8$ and $L = 30$, the search is done for $L_{eq} = 4$ to find a close-to-optimal SER curve, considering the findings of Fig. 2.

Firstly, the results of Fig. 3 suggest that a very desirable SER improvement can be achieved using linear STE and MCD on MSSK. Numerically, with respect to MCD-demodulated 8-MSSK and at a SER level of 10^{-3}, a molecule per bit (M) gain of roughly 18.5% can be achieved using one-tap linear TE, 38.4% with the one-tap one-adjacent STE defined by (12), 36.5% with simplified (12) where $\theta = \alpha\beta$, and 40.3% with a close-to-optimal linear STE with $L_{eq} = 4$.

One thing to note is that even though the *space* component of STE is introduced to remove ILI from the system, the value of β turned out to be positive, unlike α. Since α is negative, the past arrival RV at the intended antenna is subtracted from its current arrival RV. This operation results in a new RV with a smaller mean (due to subtraction) but larger variance, which decreases the $\frac{\mu}{\sigma}$ ratio (increases arrival noise) at the intended antenna's arrival count. Interestingly, for the results in Fig. 3, a positive β implies that the system is better-off by deliberately adding ILI to the intended arrival count to *restore* the received power loss due to the TE part of the STE.

V. Conclusion

In this letter, linear time and space-time equalizers for molecular-IM have been proposed. Through theoretical and computer simulation-based results, the SER improvement of equalization on molecular-IM are shown. It is found that STE outperforms TE (as it considers both ISI and ILI), whilst TE offers a lower complexity solution that allows analytical derivation of its optimal coefficient(s). Theoretical sub-optimal cost functions for STE and combining equalizers with more sophisticated detectors are considered as future works.

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