Sequency Index Modulation: A Novel Index Modulation for Delay-Sequency Domain Waveforms

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***Abstract*—In this letter, the sequency domain is introduced as a novel resource to propose a new index modulation, which is named sequency index modulation (SeIM). In SeIM, information bits are implicitly conveyed through the indices of the activated sequencies enabling bit error rate (BER), spectral efficiency, and power efficiency enhancement. Also, two strategies are proposed to allocate power among the activated sequencies revealing a trade-off between BER and power efficiency. Moreover, results are analytically derived for the BER of SeIM decoder. Finally, the recently proposed orthogonal time sequency multiplexing modulation is utilized to evaluate the performance of SeIM over doubly-spread channels.**

***Index Terms*—Index modulation, delay-sequency domain, sequency index modulation, OTSM.**

I. Introduction

INDEX modulation (IM) is utilized as a simple yet efficient technique to achieve improvement in terms of bit error rate (BER), spectral efficiency (SE), and power efficiency (PE) in wireless communications [1]. Also, further enhancement is attainable in IM designs by expanding the indexing notion to the energy and constellation domains to exploit the advantages of multiple modes and their compositions [2], [3]. Moreover, to obtain significant gain with no extra complexity in detection, index combination (IC) selection and bit-to-IC (BIC) mapping algorithms have been recently proposed being straightforward for any generalized IM schemes [4]. Such flexible attributes of IM make it promising for new candidate waveforms to provide ultra reliable and green communications in the sixth generation (6G) of wireless networks.

Orthogonal time sequency multiplexing (OTSM) has been recently proposed as a forward-looking single-carrier waveform in delay-sequency (DS) domain offering similar BER to multi-carrier orthogonal time frequency space (OTFS) modulation in doubly-spread channels (DSCs) under high mobilities; however, with much lower complexity due to the sequency orthogonality and low implementation complexity of Walsh-Hadamard transform (WHT) [5]. Thanks to the WHT, the modulation and demodulation in OTSM are done through only addition/subtraction in DS domain and no multiplication operation is needed making OTSM superior to all fast Fourier transform (FFT) based waveforms such as orthogonal frequency division multiplexing and OTFS in terms of implementation complexity. Therefore, designing a new IM based on the resources of DS domain will pave the way of low-complexity signal processing and waveform design in 6G.

In the literature, IM has been well studied in time-frequency and delay-Doppler domains by considering multiple modes and different resources such as time, frequency, modulation type, code, antenna, channel, polarization, pulse, and filter shape [1], [2], [3], [4], [6]. However, IM has not been yet investigated in DS domain and the sequency dimension has not been yet introduced as a resource for IM. Taking the advantages of sequency orthogonality in DS domain, this letter introduces the sequency domain as a novel dimension for IM and a new IM scheme is accordingly proposed in DS domain, which is named sequency index modulation (SeIM).

In the SeIM encoder, sequencies in the DS grid are first activated in an on-off keying (OOK) manner based on the majority bit value of half of the input bit-stream, which is implicitly conveyed to the receiver with no power consumption through the indices of the activated sequencies. Then, the activated sequencies are employed to explicitly convey the remaining half of the input bit-stream. As a result, the BER, SE, and PE are enhanced. Because of the simplicity of the existing coherent OOK technology for detecting the sequency status and the low complexity of OTSM, the SeIM-OTSM can be a promising candidate waveform in 6G.

Motivated by the above explanations, the contributions in this letter are summarized as follows:

i) The novel idea of SeIM is proposed and its performance gains are evaluated through the OTSM waveform over DSC.

ii) Two strategies are proposed for power allocation among the activated sequencies to retain the power originally allocated to the inactive sequencies in DS domain.

iii) Analytical closed-form formulas are derived for signal to noise ratio (SNR) and BER revealing the fact that BER and SE are traded-off against PE in SeIM.

The rest of this letter is organized as follows. The structure of SeIM encoder and decoder are described in Section II. The power allocation strategies are presented in Section III followed by the closed-form BER expressions derived in Section IV. The SE and complexity of SeIM are explained in Section V followed by the simulation results discussed in Section VI. Finally, conclusion and future insight are summarized in Section VI.

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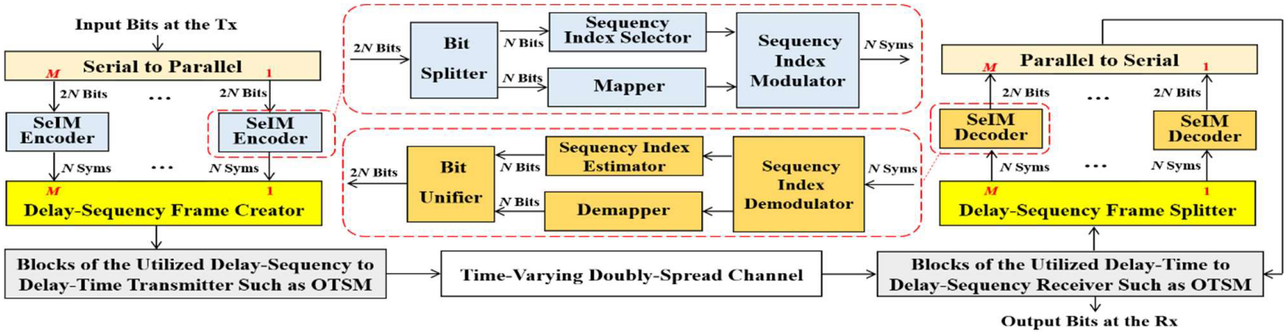

Fig. 1. The schematic of SeIM at the transmitter (Tx) and receiver (Rx).

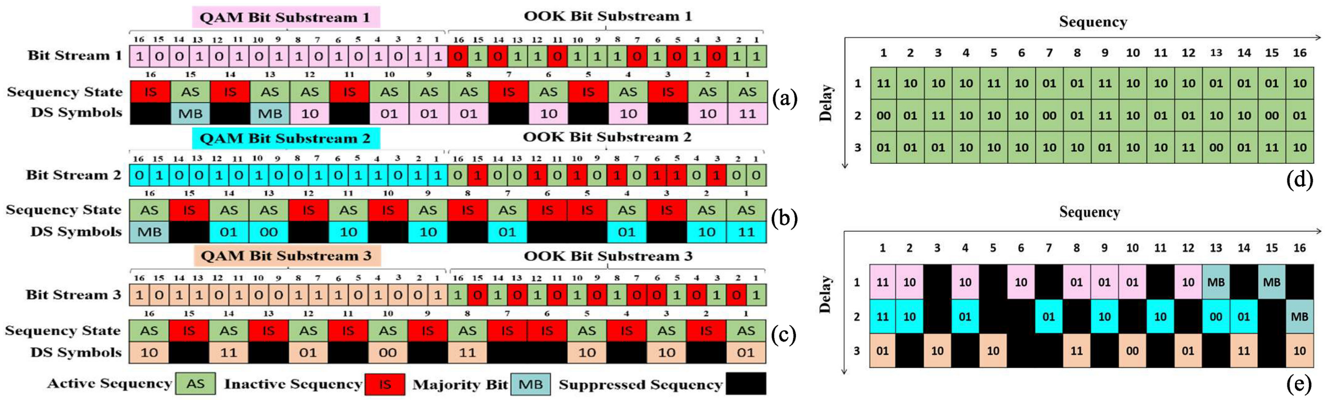

Fig. 2. Some examples of the SeIM encoder for 4-QAM, WHT size N=16, and the number of delay bins M=3, (a): The majority bit value is one in bit-stream 1, (b): The majority bit value is zero in bit-stream 2, (c): The majority bit value is one in bit-stream 3, (d): The DS representation of symbols before SeIM coding, (e): The DS representation of symbols after SeIM coding.

II. The Structure of SeIM Encoder and Decoder

In this section, the concept of SeIM is described based on the block diagram depicted in Fig. 1 and different examples illustrated in Fig. 2.

A. Bit-Substreams in SeIM

Assuming that at the transmitter, the symbols are decorated in a DS grid including M delay bins and N sequency bins, the serial input bits are first divided into M bit-streams of length $2N$, where N corresponds to the size of the WHT as well. To perform the SeIM coding, the bit-stream B of length $2N$ is divided into two bit-substreams of length N through the bit splitter shown in Fig. 1. As depicted in Figs. 2(a) to 2(c), the first and second bit-substreams being on the right and left sides of the splitting points are named OOK bit-substream (OOK-B) and quadrature amplitude modulation (QAM) bit- substream (QAM-B), respectively. The OOK-B is used to determine the indices of the activated sequencies. Also, the QAM-B includes the information, which is explicitly conveyed by the indices of the activated sequencies.

B. Grouping the Activated Sequencies in SeIM Encoder

The sequency index selector shown in Fig. 1, groups the activated and inactive sequencies based on the majority bit value [6] of each OOK-B, which is determined by the Hamming weight, i.e., the number of ones in the binary OOK-B. If the Hamming weight of the OOK-B is equal or more than $N/2$, namely half of the WHT size, the ones are majority and the majority bit value is one, as shown in Figs. 2(a) and 2(c). Otherwise, the zeroes are majority and the majority bit value is zero. Please refer to Fig. 2(b) for details of this operation.

C. Sequency Selection, Mapping, and Modulation in SeIM

As depicted in Figs. 2(a) to 2(c), the index of each bit in OOK-B corresponds to the index of each sequency. Therefore, the indices of those bits which are equal to the majority bit value are grouped as the indices of the activated sequencies and the remaining indices of the OOK-B are associated with inactive sequencies. The second functionality of SeIM encoder is mapping the second bit-substream QAM-B to the signal constellation symbols. To do so, the remaining inactive sequencies are first suppressed before the signal modulation through the sequency index modulator depicted in Fig. 1 and the QAM-B is mapped based on the modulation order. Then, the QAM symbols are modulated by the group of the activated sequencies. In this stage, significant performance gain is attainable in terms of BER through the BIC mapping principle addressed in [4] to map the QAM-B to the sequency IC obtained through the majority bit of OOK-B. The related performance gains will be discussed in Section VI.

D. Detecting the Activated Sequences at Receiver

In the SeIM encoder, some of the activated sequencies may be considered as the excess sequencies (ES). The excess sequencies are used to signal the value of the majority bit to the receiver for coherent OOK detection of the activated sequencies through the sequency index estimator depicted in Fig. 1. Then, the list of the activated sequencies and the corresponding DS frame are feedbacked to the blocks of the utilized delay-time to DS receiver such as OTSM to do other required operations in DS domain. Defining N_{ES} and N_{AS} as the number of excess sequencies and activated sequencies, respectively, we have $N_{\mathrm{ES}} = N_{\mathrm{AS}} - (\frac{N}{\log_2^Q})$ in which Q denotes the QAM order in the Q-arry QAM signal constellation. Accordingly, as shown in Fig. 2(c), when N_{ES} is equal to zero, the majority bit value is one and the signaling of the majority bit to receiver is not done since this state is predefined in the SeIM encoder and decoder.

Let E_B denote the estimation process of bit-stream B, two estimation processes are considered for the coherent OOK detector. In the first estimation process, denoted by $E_{OOK\text{-}B}$, the position of activated and inactive sequencies are estimated through the sequency index estimator block depicted in Fig. 1 so as to obtain the first bit-substream OOK-B. The second estimation process, denoted by $E_{QAM\text{-}B}$, corresponds to the demapper block depicted in Fig. 1 used to demap QAM-B. The probability of error of coherent OOK detection in SeIM will be analytically derived in Section IV.

An example of DS representation of information symbols before and after SeIM coding is presented in Figs. 2(d) and 2(e), respectively. To take the advantages of SeIM in DS domain, the output of DS frame creator depicted in Fig. 1 is used as the input for any waveform in DS domain such as OTSM. In Sections V and VI, it will be shown that SeIM-OTSM outperforms OTSM in terms of SE and BER over DSCs with high mobilities.

III. The Power Allocation Strategies in SeIM

To control the power originally allocated to the inactive sequencies, two strategies are practically attainable, which are named power reallocation (PR) and power store (PS). In the PR strategy, the power initially allocated to the inactive sequencies is evenly reallocated among the activated sequencies after suppressing the inactive sequencies. Thus, the power of each activated sequency increases. However, in the PS strategy, the initial power of the inactive sequencies is suppressed and it is stored to be allocated to the next frames.

Let $\mathcal{N}_{\mathrm{MB}}$ be a random variable representing the number of majority bit in an arbitrary bit-stream with $E(\mathcal{N}_{\mathrm{MB}}) = \mu_{\mathrm{MB}}$, where E(.) denotes the mathematical expectation. Defining P_{TF} as the transmit power originally allocated to each DS frame and σ^2 as the additive white Gaussian noise (AWGN) power per sequency, the average SNR of an activated sequency under the PR and PS strategies can be readily written as in (1) and (2), respectively:

$$\overline{\gamma}_{\mathrm{PR}} = \frac{(P_{\mathrm{TF}}/M\mu_{\mathrm{MB}})}{\sigma^2} = \frac{P_{\mathrm{TF}}}{M\mu_{\mathrm{MB}}\sigma^2}, \tag{1}$$

$$\overline{\gamma}_{\mathrm{PS}} = \frac{(P_{\mathrm{TF}}/MN)}{\sigma^2} = \frac{P_{\mathrm{TF}}}{MN\sigma^2}. \tag{2}$$

From (1) and (2), it can be concluded that $\overline{\gamma}_{\mathrm{PR}} \geq \overline{\gamma}_{\mathrm{PS}}$, because $\mu_{\mathrm{MB}} \leq N$. It is worth mentioning that for a sequence of independent equiprobable bits, i.e., an uncoded independent identically distributed (i.i.d) OOK-B, $\mathcal{N}_{\mathrm{MB}}$ becomes a binomial random variable with $E(\mathcal{N}_{\mathrm{MB}}) = \mu_{\mathrm{MB}} \approx N/2$, which results in $\overline{\gamma}_{\mathrm{PR}} \approx 2\overline{\gamma}_{\mathrm{PS}}$, i.e., PR strategy outperforms PS strategy with an SNR gain of roughly 3 dB.

IV. The Analytical BER of SeIM Decoder

In this section, the analytical closed-form BER expressions are derived for PR-SeIM and PS-SeIM decoders. Let $P_e(E_B)$, $P_e(E_{OOK\text{-}B})$, and $P_e(E_{QAM\text{-}B})$ denote the error probability for estimation processes E_B, $E_{OOK\text{-}B}$, and $E_{QAM\text{-}B}$, respectively. For the sake of simplicity in computations; however, without loss of generality, the processes $E_{OOK\text{-}B}$ and $E_{QAM\text{-}B}$ are assumed to be independent and as the OOK-B and QAM-B have the same length of *N*, they are equiprobable, i.e., $P(E_{OOK\text{-}B}) = P(E_{QAM\text{-}B}) = \frac{1}{2}$. Accordingly, $P_e(E_B)$ based on the total probability law can be readily written as

$$P_e(E_B) = P(E_{OOK\text{-}B})P_e(E_{OOK\text{-}B}) + P(E_{QAM\text{-}B})P_e(E_{QAM\text{-}B}). \tag{3}$$

In the above, the estimation of the activated and inactive sequencies through the coherent OOK detector is same as the binary amplitude shift keying detection whose error probability over an AWGN channel with the Rayleigh fading is given as

$$P_e(E_{OOK\text{-}B}) = (1 - \rho\sqrt{P_s/(\sigma^2 + \rho^2 P_s)})/2, \tag{4}$$

where ρ, P_s, and σ^2 represent the fading amplitude, average power of each symbol, and noise power, respectively [7]. Since in $E_{OOK\text{-}B}$, all activated and inactive sequencies contribute the estimation process, P_s in terms of P_{TF} is given as $P_s = \frac{P_{\mathrm{TF}}/M}{N} = \frac{P_{\mathrm{TF}}}{MN}$ and $P_s = \frac{(P_{\mathrm{TF}}/M)\times(\mu_{\mathrm{MB}}/N)}{N} = \frac{P_{\mathrm{TF}}\mu_{\mathrm{MB}}}{MN^2}$ for the PR and PS strategies, respectively. Accordingly, $P_e(E_{OOK\text{-}B})$ for an uncoded i.i.d OOK-B under the mentioned strategies can be written as

$$P_e^{\mathrm{PR}}(E_{OOK\text{-}B}) = (1-\rho\sqrt{P_{\mathrm{TF}}/(\sigma^2 MN + \rho^2 P_{\mathrm{TF}})})/2, \tag{5}$$

$$P_e^{\mathrm{PS}}(E_{OOK\text{-}B}) = (1-\rho\sqrt{P_{\mathrm{TF}}\mu_{\mathrm{MB}}/(\sigma^2 MN^2 + \rho^2 P_{\mathrm{TF}}\mu_{\mathrm{MB}})})/2. \tag{6}$$

Similar to the BER of Q-arry QAM given in [7], $P_e(E_{QAM\text{-}B})$ can be written as

$$P_e(E_{QAM\text{-}B}) = 2\alpha(1 - \lambda - \alpha/2 + \alpha\lambda\tan^{-1}(1/\lambda)/2\pi), \tag{7}$$

where, $\alpha = (\sqrt{Q} - 1)/\sqrt{Q}$, and $\lambda = \rho\sqrt{3P_s/(\sigma^2(Q-1) + 3\rho^2 P_s)}$.

Here, P_s denotes the average power of each Q-arry QAM symbol. Unlike the first estimation process $E_{OOK\text{-}B}$ in which all *N*sequencies are used, just μ_{MB} activated sequencies are utilized in the second estimation process $E_{QAM\text{-}B}$. Therefore, P_s can be obtained by dividing the power allocated to each delay bin namely P_{TF}/M by the number of the activated sequencies. Hence, P_s is given as $P_s = \frac{P_{\mathrm{TF}}/M}{\mu_{\mathrm{MB}}} = \frac{P_{\mathrm{TF}}}{M\mu_{\mathrm{MB}}}$ and $P_s = \frac{(P_{\mathrm{TF}}/M)\times(\mu_{\mathrm{MB}}/N)}{\mu_{\mathrm{MB}}} = \frac{P_{\mathrm{TF}}}{MN}$ for the PR and PS strategies, respectively. Accordingly, $P_e(E_{QAM\text{-}B})$ for an uncoded i.i.d QAM-B under mentioned strategies can be written as

$$P_e^{\mathrm{PR}}(E_{QAM\text{-}B}) = 2\alpha(1 - \lambda_{\mathrm{PR}} - \alpha/2 + \alpha\lambda_{\mathrm{PR}}\tan^{-1}(1/\lambda_{\mathrm{PR}})/2\pi)/\log_2^Q, \tag{8}$$

TABLE I
SPECTRAL EFFICIENCY COMPARISON

M	64	128	256	512	1024
N	64	128	256	512	1024
$l_{\max}$	2	5	11	21	43
l_{ZP}	5	11	23	43	87
μ_{MB}	36	101	212	421	835
$\eta_{\mathrm{OTSM}}/\log_2^Q$	0.921	0.914	0.910	0.916	0.915
$\eta_{\mathrm{SeIM\text{-}OTSM}}/\log_2^Q$	1.440	1.635	1.663	1.669	1.661

$$P_e^{\mathrm{PS}}(E_{QAM\text{-}B}) = 2\alpha(1-\lambda_{\mathrm{PS}}-\alpha/2+\alpha\lambda_{\mathrm{PS}}\tan^{-1}(1/\lambda_{\mathrm{PS}})/2\pi)/\log_2^Q, \quad (9)$$

where, $\lambda_{\mathrm{PR}} = \rho\sqrt{3P_{\mathrm{TF}}/(\sigma^2(Q-1)M\mu_{\mathrm{MB}}+3\rho^2P_{\mathrm{TF}})}$ and $\lambda_{\mathrm{PS}} = \rho\sqrt{3P_{\mathrm{TF}}/(\sigma^2(Q-1)MN+3\rho^2P_{\mathrm{TF}})}$.

Putting (5) and (8) into (3) and also by substituting (6) and (9) into (3), the overall BER closed-form formulas for the proposed SeIM decoder are obtained as

$$P_e^{\mathrm{PR}}(E_B) = \frac{(1-\rho\sqrt{P_{\mathrm{TF}}/(\sigma^2MN+\rho^2P_{\mathrm{TF}})})}{4} + \alpha(1-\lambda_{\mathrm{PR}}-\alpha/2+\alpha\lambda_{\mathrm{PR}}\tan^{-1}(1/\lambda_{\mathrm{PR}})/2\pi)/\log_2^Q, \quad (10)$$

$$P_e^{\mathrm{PS}}(E_B) = (1-\rho\sqrt{P_{\mathrm{TF}}\mu_{\mathrm{MB}}/(\sigma^2MN^2+\rho^2P_{\mathrm{TF}}\mu_{\mathrm{MB}})})/4 + \alpha(1-\lambda_{\mathrm{PS}}-\alpha/2+\alpha\lambda_{\mathrm{PS}}\tan^{-1}(1/\lambda_{\mathrm{PS}})/2\pi)/\log_2^Q. \quad (11)$$

V. THE SPECTRAL EFFICIENCY AND COMPLEXITY OF SEIM

Assuming that the SE of a frame with no SeIM in DS domain is ideally given as $\eta = \log_2^Q$ bps/Hz, the SE after SeIM is obtained as $\eta_{\mathrm{SeIM}} = \log_2^Q(1+\mu_{\mathrm{MB}}/N)$ bps/Hz. Also, the SE of OTSM is given as $\eta_{\mathrm{OTSM}} = \log_2^Q(1-l_{\mathrm{ZP}}/M)$ bps/Hz, where l_{ZP} denotes the length of zero-padded (ZP) symbols in OTSM constrained as $l_{\mathrm{ZP}} \geq 2l_{\max}+1$ to prevent from inter block interference, where $l_{\max}$ represents the maximum delay index of the related DSC. Accordingly, the SE of SeIM-OTSM can be readily written as $\eta_{SeIM-OTSM} = \log_2^Q(1-l_{\mathrm{ZP}}/M)(1+\mu_{\mathrm{MB}}/N)$ bps/Hz. In Table I, the SE of SeIM and SeIM-OTSM are shown for different values of DS domain and SeIM parameters revealing the significant gain of SeIM in terms of SE.

The complexity of SeIM decoder corresponds to the complexity of the comparison between the received bits and the majority bit over each delay bin done through *MN* complex additions and the complexity of thresholding in coherent OOK carried out through *MN* complex additions. Thus, the total complexity of SeIM is in the order of $\mathcal{O}(2MN)$ and no multiplication is needed making SeIM superior to FFT-based IM schemes. Also, the complexity of detection corresponds to the complexity of the related channel estimator and detector used within the utilized waveform in DS domain such as OTSM [5], which decreases since instead of N sequencies, N_{AS} sequencies are used for the related calculations in SeIM-OTSM. For example, the complexity of channel estimation in SeIM-OTSM decreases from $\mathcal{O}(MNL)$ needed in OTSM to $\mathcal{O}(MN_{\mathrm{AS}}L)$, where L is equal or less than the number of dominant reflectors in the DSCs [5].

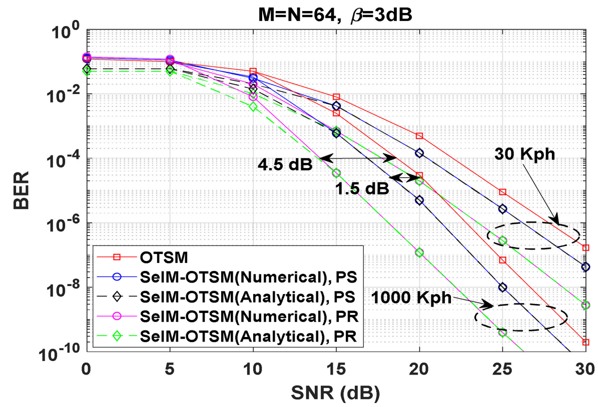

Fig. 3. The BER performance for 4-QAM uncoded data with varying mobility speed of 30 Kph and 1000 Kph under PS and PR strategies.

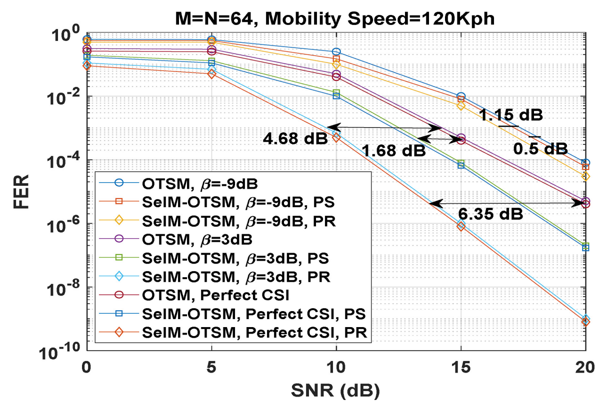

Fig. 4. The FER performance for 4-QAM coded data and varying β.

VI. SIMULATION RESULTS

In the following simulation studies, the carrier frequency, available bandwidth, and maximum delay spread are 4 GHz, 10 MHz, and 4 μs, respectively. The iterative detector based on the Gauss Seidel method is used in OTSM [5]. The standard extended vehicular A channel delay model is adopted with the power delay profile of [0, −1.5, −1.4, −3.6, −0.6, −9.1, −7, −12, −16.9] in dB and the tap delay of [0, 30, 150, 310, 370, 710, 1090, 1730, 2510] in ns. Also, fractional delay and Doppler with independent uniform Doppler shifts are considered at different speeds over each path.

The BER performance of OTSM and SeIM-OTSM under PS and PR strategies are depicted in Fig. 3 for different mobility speeds. As illustrated in Fig. 3, SeIM-OTSM surpasses OTSM under both PS and PR strategies for different mobility speeds. For instance, compared to OTSM and for a target BER of 10^{-4}SeIM-OTSM approximately results in SNR gain of 4.5 dB and 1.5 dB for PR and PS strategies, respectively. Also, this gain stays almost constant as SNR increases.

In Fig. 4, the frame error rate (FER) performance of OTSM is compared with SeIM-OTSM under PS and PR strategies for

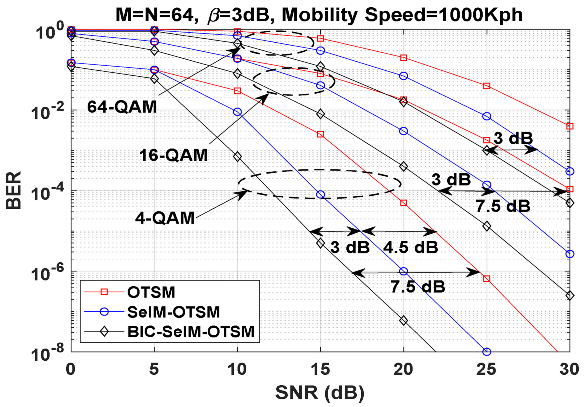

Fig. 5. The BER performance of OTSM, SeIM-OTSM, and BIC-SeIM-OTSM for different QAM orders.

perfect channel state information (CSI) and different cases of excess pilot power β. The turbo iterative low-density parity check (LDPC) decoder [5] of 1/2 rate is used with an LDPC codeword length of 3840 bits. Observing Fig. 4, it is concluded that SeIM-OTSM outperforms OTSM for both perfect CSI and pilot-based channel estimation with imperfect CSI. For example, compared to OTSM and for a target FER of 10^{-3}, SeIM-OTSM with PR strategy roughly results in SNR gain of 1.15 dB, 4.63 dB, and 4.68 dB for $\beta = -9$ dB, $\beta = 3$ dB, and perfect CSI, respectively. Also, under PS strategy, SNR gain of 0.5 dB, 1.62 dB, and 1.67 dB are obtained for$\beta = -9$ dB, $\beta = 3$ dB, and perfect CSI, respectively. Thus, $\beta = 3$ dB is sufficient to obtain a performance gain same as perfect CSI under both PS and PR strategies. The further performance gain in high SNR values in Fig. 4 compared to Fig. 3 is due to the turbo iterative LDPC coding.

From the comparison of results in Figs. 3 and 4, it is concluded that the SNR gain for PR strategy is 3 dB better than that of PS strategy. This is quite consistent with $\overline{\gamma}_{\text{PR}} \approx 2\overline{\gamma}_{\text{PS}}$ analytically derived in Section III. Also, the analytical results obtained for the coherent OOK detector in DS domain closely match the numerical results for SNR values more than 15 dB, which is acceptable in practice. The mismatch in results for low SNR values is due to the independency assumed between $E_{OOK\text{-}B}$ and $E_{QAM\text{-}B}$ to simplify the calculations in obtaining (10) and (11); however, wrong estimation of OOK-B results in the probability of error in estimating QAM-B in low SNR values. The cause of significant gain of 4.5 dB obtained for PR strategy compared to the marginal gain of 1.5 dB for PS strategy can be attributed to the power increase of the activated sequencies in PR strategy enhancing the quality of detection at the receiver. However, the PE of SeIM-OTSM compared to OTSM for PS strategy is $10log_{10}^{N/\mu_{\text{MB}}}$ dB better than that of PR strategy since μ_{MB}/N of the power is merely utilized in PS strategy over each delay bin. Therefore, adopting SeIM with OTSM always gives rise to the SE and BER enhancement; however, depending on the power allocation policy, SE and BER are traded-off against PE.

In Fig. 5, the BER performance of SeIM-OTSM for different QAM orders are compared with BIC-SeIM-OTSM in which after listing the activated sequencies through the majority bit of OOK-B, i.e., specifying the sequency IC for each delay bin, the remaining QAM-B is mapped through the BIC mapping principle presented in [4]. Although finding the best mapping in BIC-SeIM-OTSM imposes a complexity order of $\mathcal{O}(2^{\lfloor \log_2(C_N^{\mu_{\text{MB}}}) \rfloor^2})$ to the SeIM encoder, BIC-SeIM-OTSM outperforms SeIM-OTSM and OTSM by 3 dB and 7.5 dB, respectively, where C_m^n is the binomial coefficient and $\lfloor \cdot \rfloor$ denotes the floor operator. Also, this gain is almost constant as the SNR and QAM order increase.

VII. Conclusion and Future Insight

The sequency orthogonality in DS domain has been innovatively exploited to introduce the sequency dimension as a new resource for IM and the novel concept of SeIM has been proposed as a promising frame structure to achieve enhancement in terms of BER, PE, and SE in DS domain. Analytical studies and simulation results have shown that depending on the power allocation strategy utilized in SeIM, the BER and SE are traded-off against PE. Moreover, SeIM-OTSM outperforms OTSM with SNR gain of 4.5 dB and 1.5 dB for PR and PS strategies, respectively. Thanks to the BIC mapping principle in [4], further performance gain of 7.5 dB is obtained by BIC-SeIM-OTSM in DSCs with high mobilities.

In this letter, we assumed that estimation of OOK-B is independent from QAM-B verified as an acceptable assumption in high-SNR regime; however, it may lead to bit error propagation and burst error in low-SNR values. The motivation of making OOK-B and QAM-B independent in decoding is that instead of each bit from OOK-B encoded in a single sequency state, it can be encoded in the states of two consecutive sequencies, i.e., a sequency pair. The mentioned idea aims at avoiding the bit error propagation in SeIM whilst retaining the benefits of concept, left as the future work.

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